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# CONTENT LEADERSHIP IN MATHEMATICS EDUCATION: A LITERATURE REVIEW

Dragana Martinovic, Heidi Horn-Olivito, and Nesreen El Kord[1]

Education is an ever-evolving field. Over the last few decades, ideas about mathematics education have changed in multiple ways in order to accurately reflect developmental, learning, and motivational theories indicating that all children are capable learners and deserve optimal educational opportunities. Canadian mathematics curricula and pedagogies continue to be influenced by the development of new understandings of how children learn and grow. Although mathematical concepts taught in the classroom have ancient roots, today's classroom practices focus on building mathematical proficiency and deep conceptual understanding in authentic ways. In efforts to provide support to mathematics teachers in Canadian schools, appointment of school-based leaders began to take place in some parts of the country over the past decade or so.

In this report, excerpts and recommendations from current research addressing various aspects of excellence in teaching, pedagogical and content knowledge, teacher leadership, and overall school improvement are provided. Issues discussed in the report include venues for improving mathematics instruction, student engagement, and content knowledge. The report begins by defining governing terms and concepts as it presents an emerging model of mathematics teacher leadership. A deeper look into research findings lays the grounds for some further recommendations.

[1] This literature review was conducted by the Mathematics Leadership Community of Practice research team.

# Quality Teaching

Joseph Rogus (1988) describes effective classroom teachers as:

> reflective practitioners who know the research and the literature on teaching; they model the best practice in instruction; they are well grounded in their discipline(s) and are liberally educated; they place their classrooms in a larger social context and understand alternative visions of school and how external political and cultural factors influence these variables; they demonstrate command of program regularities; and they have internalized the wisdoms of daily practice. (p. 48)

Teachers' beliefs and personal qualities impact their teaching practices. Ferguson and Danielson (2014) emphasize that providing quality teaching in the classroom is important for keeping students engaged and motivated to learn. For Ferguson and Danielson, quality education implies providing both (a) press and (b) support, where *press* is "keeping students busy and on task and pressing them to think rigorously and persist in the face of difficulty" (p. 100), and *support* is providing "caring teacher-student relationships, captivating lessons and other practices that students perceive as supportive" (p. 100).

In their study, Ferguson and Danielson (2014), grouped homeroom teachers using four categories—based on (a) ability to connect with colleagues (active vs. isolated) and (b) perceived expectations of students (believers vs. agnostics); see Figure 1. They found that students responded better to teachers who: were active believers, were well connected to colleagues, and had high expectations of students, than those who were isolated agnostics. Therefore, Ferguson and Danielson recommended that special attention be paid in PD attempts to strengthening collegial relations and neutralizing teacher expectations,

> Take special steps to identify, cultivate, and retain active believer teachers and create opportunities for their beliefs and behaviours to influence other teachers….make special efforts to identify isolated agnostics and provide them with both support and press to improve. (p. 361)

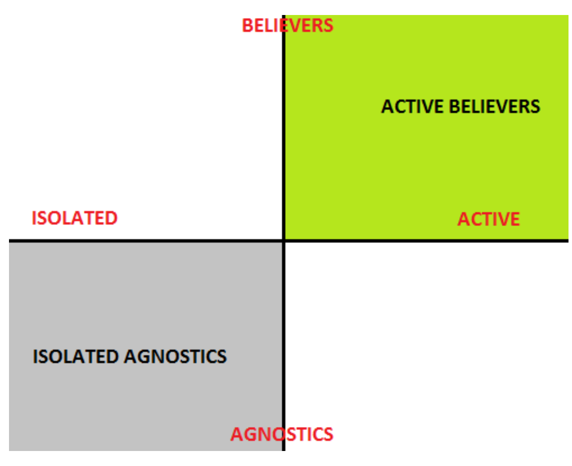


Figure 1. Visualization of four types of teachers.

Furthermore, using various sources of data collected in schools, with teachers, students, and from government sources, Ferguson and Hirsch (2014) attempted to understand how school working conditions transfer into student outcomes; considering teacher beliefs, behaviours, and quality of teaching. They found a number of factors including: manageable duties within the work scope, supportive school leaders, effective PD, and teachers' professional and pedagogical skills, to contribute to creating conditions for effective teaching.

## Teacher Leadership

Teacher leaders are first and foremost effective teachers; who are able to demonstrate on a daily basis the competencies associated with effective classroom instruction. These individuals are the innovators, problem solvers and lead-learners in their schools. According to Crowther, Ferguson, and Hann (2009) teacher leaders,

- Convey convictions about a better world by articulating a positive future for all students;
- Facilitate communities of learning by encouraging a shared, schoolwide approach to core pedagogical processes;
- Strive for pedagogical excellence by showing genuine interest in students' needs and well-being;
- Confront barriers in the school's culture and structures by standing up for children, especially disadvantaged and marginalized individuals and groups;
- Translate ideas into sustainable systems of action by internal and external networking; and
- Nurture a culture of success by acting on opportunities to emphasize accomplishments and high expectations. (p. 3)

Further, Even (1999) emphasizes the complexity of teacher leadership by stating that for successfully helping "others develop their teaching", it is not enough that one is recognized as an experienced and excellent teacher. These individuals need to have solid: (a) content and pedagogical knowledge as they stay informed on current views of mathematics teaching and learning; (b) leadership and mentoring knowledge and skills for working with peers; and (c) are able to make connections within the education community. Even also suggests that experience in conducting action research/collaborative inquiry can be an asset to such individuals.

Moreover, Cooper, Stanulis, Brondyk, Hamilton, Macaluso, and Meier (2016) observed 11 teacher leaders while describing their practices in schools. During the study, the researchers began to question their own assumptions about the background knowledge of teacher leaders who were identified by principals.

Cooper et al. recognized a critical need for teacher leaders to possess substantial knowledge base of instructional leadership and strategies for leading change. They recommended that teacher leaders need to: (a) understand the nature of relationships governing professional learning communities, mentoring, and the types of collaborative practices that support teacher learning, and (b) understand organizational change and strategies for driving change among their peers.

## Positioning Mathematics Leaders

Mathematical proficiency is a goal that mathematics education has for every student, while daily engaging students in mathematics learning intended to guide them toward new understandings and connections. It is therefore no surprise that this aim rests squarely on the quality of experience students have in learning mathematics. Following this line of thought, dedicated educators, who deeply understand mathematics content and pedagogy, intentionally construct rich learning experiences that engage and inspire their students. Building the expertise of all educators to reach this level of dedication, requires the support and action of both school and system leaders of mathematics education.

How leadership is generally viewed determines the set of actions taken in the process of supporting school teachers, as Lambert (2003) posits, "How we define leadership defines how we participate in it" (p. 4). Exploring roles and responsibilities of leaders within literature on mathematics education leadership, we examined aspects from different teacher leadership frameworks, in conjunction with content knowledge literature, in an attempt to create a holistic picture of mathematics teacher leadership model, (see Figure 2).

Borko et al., (2014), posit that leaders of mathematics PD utilize *Mathematical Knowledge for Professional Development (MKPD)*, which encompasses—"specialized content knowledge, pedagogical content knowledge, and learning community knowledge [which] go beyond and look different than the knowledge [of] a typical mathematics classroom teacher... [Also,] PD leaders should be knowledgeable about how to work productively with adult learners, and construct environments for teachers to collaborate about relevant topics" (p.165).

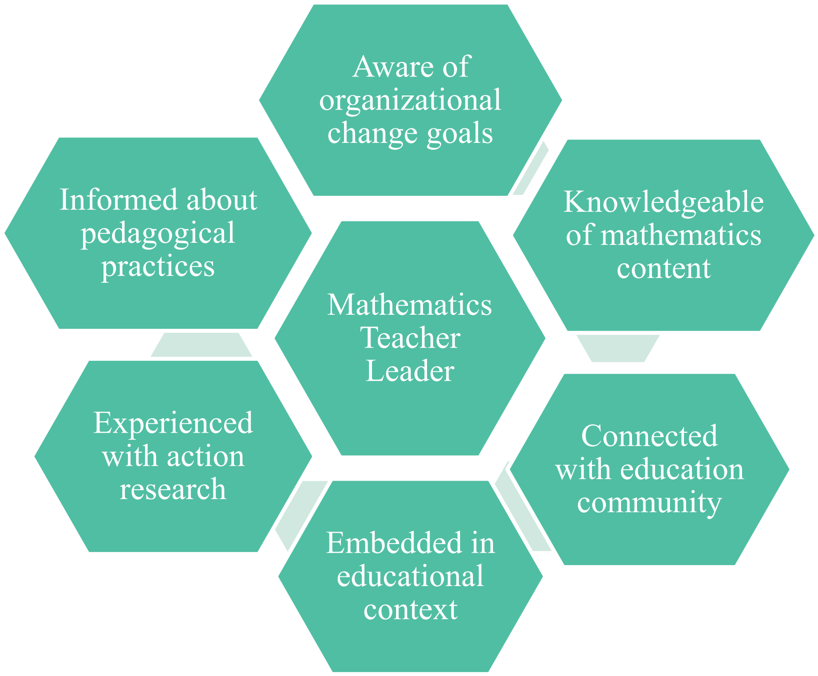


Figure 2. An emerging model of mathematics teacher leadership--working with adults to benefit students.

## Models of Professional Development (PD)

A number of studies focused on mathematics PD; in an effort to examine the effectiveness of current models of mathematics teacher leadership and coaching. A number of such studies is discussed here for the purpose of providing a view of existing models and resulting recommendations.

Elliott, Kazemi, Lesseig, Mumme, Carroll, and Kelley-Petersen (2009) conducted a research and development project, which studied how leaders learn to cultivate mathematically rich PD activities. In the era when increasing numbers of teachers are asked to take upon leadership roles in which they may be expected to occasionally (or predominantly) lead PD events, this project intended to fill the gap in understanding how best to support these teachers. The researchers developed a series of seminars for teacher leaders using two frameworks: (a) *sociomathematical norms* (norms for mathematical reasoning) and (b) *a set of practices for organizing productive mathematical discussions*. The seminars involved watching videos, which featured a facilitator leading mathematics PD and discussions around how the facilitator's approach fits the two frameworks. This approach attempted to help leaders gain skills for working with mathematics teachers (e.g., to know how to tease out mathematical details that are evident or missing from the participants' discussions; to deal with participants' positioning towards the subject or the school system organization; to know how to tactfully approach eventual participants' misconceptions or lack of understanding of mathematical concepts).

During the PD sessions, the leaders in Elliott et al.'s study, needed to develop a skill of prompting their colleagues to explain and justify their mathematical reasoning, thus "[u]nderstanding how to navigate the fine line between being a colleague and facilitating learning was central to leaders' sense making of sociomathematical norms" (p. 373). To overcome the limitations in teachers' positioning towards mathematics (e.g., "I was never good in math") or distancing from certain topics/examples based on grade level they teach, the leaders "focused on identifying mathematical ideas worth developing that are equally important for teacher learning regardless of grade level or self-identified strength or weakness" (p. 374). During the first two years of the study, Elliott et al. discovered that leaders need to get better acquainted with a third framework—the *mathematical knowledge for teaching* (MKT; Ball, Thames, & Phelps, 2008). They emphasized a need for the session tasks to: (a) focus on teacher reasoning rather than the solution of a mathematics problem (focus on specific knowledge for teaching a topic); (b) utilize examples of errors and misconceptions; and (c) have clear but flexible goals, so that they could reframe the tasks and use the tools according to teachers' solutions.

Moreover, from the same project, Jackson, Cobb, Wilson, Webster, Dunlap, and Appelgate (2015) developed a coding scheme for assessing mathematics leaders' development of focal practices; to gather evidence relevant to each of the three goals for their learning (i.e., treating teacher learning as a progression, designing supports for teachers' learning, and pressing on teachers' ideas; see Table 1). Each mathematics leader position towards PD practice was categorized as stagnant or progressive, with an obvious intent to see the improvement in their stance. Also, Jackson et al. added another goal, which is to change leaders' assumptions about how teachers develop new practices and to highlight their role in scaffolding development of teachers for whom they organize PD sessions.

Table 1. Evidence of Improvements in the Mathematics Leaders' Practice, Jackson et al. (2015)

| Abbreviated goals for math leaders' learning | Evidence of stasis | Evidence of improvement |
|---|---|---|
| Treating teacher learning as a progression | – Suggests teachers' practices could be rectified in an isolated activity or session;<br>– Uses transmission model when designing or enacting activities;<br>– Designs disconnected activities and PD sessions. | – Considers instructional improvement as a progression which needs extended, sustained support;<br>– Makes connections between sessions and activities. |
| Designing supports for teachers' learning | – Designs PD without considering teachers' current practices;<br>– Has difficulty formulating learning goals for PD activities;<br>– Makes weak connections with mathematics instruction. | – Differentiates PD according to teachers' current practices;<br>– Can formulate learning goals for PD activities;<br>– Makes strong connections with core of mathematics instruction. |
| Pressing on teachers' ideas | – Does not differentiate between teachers' ideas that are worth pursuing and others that are not;<br>– Does not make connections between them. | – Follows up on teachers' ideas appropriately and makes connections involving teachers in the process. |
| Knowing how teachers develop new practices | – Seeks to demonstrate whole activity, assuming teachers will pick up main points and enact them in their practice. | – Understands that educator's development of new practices requires unpacking and scaffolding current practices. |

Borko, Koellner, and Jacobs (2014) conducted a multi-year design research project, in which they investigated the scalability and sustainability of the *Problem-Solving Cycle (PSC)* model of mathematics PD. Due to an urgent need to prepare novice PD facilitators to successfully make use of newly developed PD models that offer high-quality learning opportunities for teachers, this study particularly examined the degree to which the *PSC*, which "is an iterative, long-term approach to mathematics PD" (p. 151), could be implemented with quality by novice local facilitators through analysis of their facilitation in workshops conducted at their schools.

The key characteristics of the *PSC* are that: (a) the PD program is ongoing, long-term, and adaptive to participants' needs and priorities; (b) communities of practice play a central role in determining what and how people learn; (c) PD activities are situated in teachers' classroom instruction through tangible artifacts of practice, particularly video; (d) the PD aims to improve content knowledge in a specific domain; and, (e) the PD focuses on student thinking and provides teachers with opportunities to make connections to their own instructional practice.

The *PSC* entails multiple cycles of three interconnected PD workshops, all organized around a rich mathematics task. In Workshop 1 of a given cycle, teachers collaboratively solve the selected mathematics task and develop plans for teaching it. After the first workshop, teachers implement the problem with their own students and their lessons are videotaped. The facilitators then select video clips that highlight key moments in the instruction and in students' thinking about the problem. Workshops 2 and 3 of the cycle focus on the teachers' classroom experiences and rely heavily on the selected video clips.

The Borko et al.'s study included 2½ years of preparation and support for teacher leaders (TLs) to facilitate the *PSC* with the mathematics teachers in their schools. The research incorporated video, classroom artifacts, and interviews to document the preparation and support provided to the TLs; and the range and quality of their implementation of the PSC.

Borko et al. concluded that developing a large number of skillful and capable PD facilitators, is key to mathematical PD. Additionally, "there is an urgent need to identify the types of knowledge and skills these facilitators draw on, and to determine how they can best be supported to develop and expand their relevant knowledge and skill" (p. 165), see Table 2.

Table 2. Ratings of different components of the *PSC* Program, Borko et al. (2014)

| Workshop Culture | Specialized Content Knowledge | Pedagogical Content Knowledge |
|---|---|---|
| Very high ratings across TLs | Workshops rated highly on teachers' specific content knowledge | TLs success in selecting video clips, relevant to teachers, and fostering a level of trust within their groups |
| Very high ratings across PSC cycles | High ratings on teachers' multiple representations and solution strategies | TLs less successful, in promoting deep analysis of instructional practices or student thinking |
| TLs success in garnering a climate of respect and promoting collaborative, collegial working relationships | Lower ratings on reasoning analysis, discussion of relationships among representations, and affordances and constraints of representations | |
| Lower ratings of participants' willingness to share ideas and take intellectual risks | | |

Koellner and Jacobs (2014) further reported on findings from the Borko et al.'s (2014) study, and presented this mathematics PD model as adaptive to goals, resources, and circumstances of the local context. Their focus was on the impact of the processes through which teachers learn, while participating in PD situated in classroom practice on teachers' knowledge and instructional practices, as well as on students' achievement over time as compared to specified models. There is a growing variety of PD programs that fall in various places along the adaptability continuum; therefore, teachers can be offered a range of learning opportunities using different formats. Koellner and Jacobs discussed adaptability of mathematical PD programs to inform policy and decision makers about which models to invest in and utilize.

Researchers from the Learning Mathematics for Teaching project developed a series of instruments to assess teachers' *MKT (Mathematical knowledge for teaching)*, including their content and pedagogical content knowledge. In the iPSC project, parallel forms of the *MKT–Middle School (MKT-MS)* instrument, designed specifically to assess the middle school mathematics teachers' knowledge of number concepts and operations. Pre- and post-program administrations of the *MKT-MS* were conducted to document changes in teachers' *MKT* over the course of their participation in the study. Additionally, an observation protocol created by the LMT researchers called the *Mathematical Quality of Instruction (MQI)* instrument (Hill, Ball, & Schilling, 2008), was used. TLs and case study teachers were videotaped twice during each semester that they participated in the project. During the fall and spring semesters one PSC lesson and one non-PSC or "typical" lesson were also videotaped. During Year 2, 52 lessons from the TLs and case study teachers were videotaped.

In gauging the impact of the PD on students' achievement, the percentage of students who scored at a proficient or advanced level on the CSAP within each of the 5 years of data collected by Koellner and Jacobs' (2014), project were considered. Students of the participating teachers were compared with students of middle school mathematics teachers in the same district, who did not participate in the iPSC, and the students at the same grade levels across the state, see Table 3.

Koellner and Jacobs (2014) concluded that participation in the PSC model of PD can support at least modest improvements in teachers' knowledge and classroom instruction, within a relatively short time frame.

Table 3. Evidence of the PD's Impact, Koellner and Jacobs (2014)

| Impact on Teachers' Mathematical Knowledge | Impact on Instructional Practices | Impact on Students' Achievement |
|---|---|---|
| Significant gain in mathematics knowledge for teaching | – TLs consistently improved on:<br>▪ working with students and<br>▪ identifying errors and imprecision<br>▪ ability to remediate student errors in both PSC lessons and typical lessons<br>– Then eventually went back to original practices.<br>– Overall increase in ratings for both TLs and case study teachers on:<br>▪ MQI<br>▪ MKT | Highest average CSAP scores attained by students taught by participating teachers |

Margolis and Doring (2012) conducted a two-year research study, investigating the organizational and social structures that allow teacher leaders to have the most positive impact on teacher PD. A movement toward embedding teacher learning in the actual work of teaching, has led to more PD in face-to-face settings and embedded in classroom contexts. In such settings, teacher leaders have been increasingly used as coaches, staff developers, and instructional leaders. To answer the call for more targeted studies of teacher leadership, this study explored one model of structured classroom-centered teacher PD: the studio classroom and a specific role (*hybrid teacher leaders—HTLs*), "An HTL is a teacher whose responsibilities include both teaching K-12 students and leading teachers in some capacity" (p. 861). Margolis and Doring define the studio classroom in relation to two levels of teacher leader modeling in schools: (a) *Level 1 modeling,* involves direct observation of teacher leader teaching-practice with students present (in which case a teacher leader's own classroom becomes a "studio" or "lab" classroom), while (b) *Level 2 modeling,* includes instances where teacher leaders openly share examples of student work, strategies, and lesson plans, as well as their pedagogical struggles and triumphs. Students are not present during Level 2 modeling.

Three *HTLs* were each individually interviewed twice, once prior to the beginning of the academic year and once at the end of the academic year. The goal was to understand how they perceived their role, including the ways in which they planned to connect their own classroom teaching to the learning of other teachers. Additionally, all three *HTLs* participated in a focus group and each was observed throughout the academic year 10 or 11 times, including two or three observations of their own classroom teaching. Typical observations included participation in or facilitation of PD events, visitations to other classrooms, leadership planning meetings, and teaching events with their own students. Additionally, the researchers observed the scheduled studio classroom events and included the *HTLs*' perceptions of their relationship to the larger school reform efforts. The study identified both benefits and limitations of the studio classrooms (see Table 4).

Table 4. Benefits and Limitations of the Studio Classrooms, Margolis and Doring (2012)

| Roles/reactions | Benefits of the Studio Classroom | Obstacles to the Studio Classroom |
|---|---|---|
| Administrators' reactions to studio classroom | – Success of studio classroom modeling linked to relationships HTLs built with fellow teachers, which created comfortable and respectful environment needed to engage in classroom inter-visitations<br>– Teachers who taught less in their own classroom would be able to model more lessons | – Concerned that HTLs' multiple roles left them little time for doing demo lessons<br>– Lack of direction at the district level left details to be worked out at school level<br>– Principals demonstrated less support for teacher leader experimentation and level 1 modeling<br>– More support in words than in action for the studio classroom<br>– Lack of direction from administrators on expectations related to focus and frequency of teacher leader modeling |
| Teacher leaders' reactions to studio classroom | – Liked having approaches modeled for them<br>– Understood better both the "why" and "how" of schoolwide reform goals<br>– Desired seeing new approaches modeled in order to integrate them more deeply into their teaching schemata | – Did not like being visited themselves<br>– Made some teachers too dependent on HTLs<br>– Were concerned about guest-teaching in other classes<br>– Did not feel it was worth the time<br>– Seemed to be asking for concrete examples of reforms-in-action<br>– Perceived a lack of systemic readiness to facilitate teacher learning through the studio model<br>– Feared of judgment<br>– Pointed to school culture barriers: fear, distrust, and privacy |
| Organizational/structural/ cultural barriers | | – Lack of compatibility of schedules for HTLs and teachers with whom they worked<br>– Little to no impact on larger school reform efforts or individual teacher learning<br>– Lack of time, money, as well as protocols and focus |

Margolis and Doring found a diminished sense of understanding and appreciation for the teacher learning process leaving logistical, social, and cultural barriers overwhelming any studio classroom implementation attempts—and teacher leaders ultimately failed to open up their classroom doors as intended.

Furthermore, Knapp (2017) describes how she, a middle school mathematics teacher, learned to enact leadership through an informal role as a teacher leader and a researcher. Her research project aimed at gaining a better understanding of the leadership development process and exploring the effect that development had on her as a middle school mathematics teacher. During the process of narrative journaling, this self-study focused on Knapp's broadening perspective and sense making about leadership during one school year. Ultimately, she uncovered the factors and experiences that supported or hindered her development as a teacher leader.

Knapp describes that she was asked to be part of a grant-funded professional learning opportunity to spend three weeks during the summer, for four years, taking mathematics content courses and engaging in leadership seminars. She was expected to bring her learning back to her colleagues, a task that she was not too comfortable doing, as she herself barely felt adequate "trying on" some of the new teaching approaches she learned about.

Shortly after the previous grant opportunity ended, she was asked by the district to serve as the "Math Studio" teacher for another grant-funded PD project that aimed to develop Math Studio classrooms. Math Studio is a model of PD that is "intensive, ongoing, and tightly connected to teaching practice. It focuses on students' mathematics learning, provides support for teachers during the rehearsal of challenging aspects of teaching, and focuses on the development of strong working relationships among teachers" (p. 252). Being the Math Studio teacher meant allowing others to observe Knapp's teaching practice while she taught her students. She was involved in the district's Math Studio professional development for two years.

Then, she was asked to join a group of 10 teachers for a third grant aimed at developing teachers as leaders. The grant concentrated on developing teacher leaders also within the context of Math Studio. The intent was that participating teachers would continue to be classroom teachers while facilitating the professional growth of colleagues, and thus themselves develop as coaches and leaders.

Through journaling, Knapp sought to communicate details about how she came to perceive and understand teacher leadership during her first of three years in the third grant-funded project. For the research, Knapp used personal journals and focus group interviews with participating teacher leaders as sources of data.

The results of this autoethnographic study reveal how Knapp's view of leadership changed during the year of the study. As her leadership identity transitioned, the researcher found that adopting a lead-by-example and lead-learner stance supported her work with colleagues. Factors that supported teacher leadership, as well as factors that hindered the process of becoming a teacher leader, were discussed and documented (see Table 5).

Table 5. Factors that support teacher leadership vs. factors that hinder the process of becoming a teacher leader, Knapp (2017)

| Factors and experiences supporting leadership | Factors and experiences hindering leadership |
|---|---|
| – maintaining a disposition of continuous learning | – confusion about leadership role |
| – developing a community of practice with colleagues | – navigating the middle ground between colleague and leader |
| – developing a system view and focusing on "big picture" | – lack of effective communication with administration |

Findings of this study suggest a number of important changes should be made in schools to identify supports and structures that assist in advancing the leadership work of teachers:

- Supporting teachers within a context of a community of practice to provide learning opportunities that would engage teachers in a shared examination of their teaching practice;
- Creating a clear and open path to teacher leadership within schools by collapsing traditional hierarchical schools can support more teachers in engaging in leadership actions;
- Administrators' support greatly impacts the development of teacher leaders within their district and schools; and
- It is crucial that the fears of emergent teacher leaders are recognized and handled proactively in a safe environment by both teachers and administrators.

Finally, Knapp suggests that teacher leaders take that first step beyond their classrooms through: (1) Adopting an attitude of continuous learning; (2) enlisting colleagues and learning together; (3) partnering with principals; (4) finding their voice; and (5) understanding the process is not easy, but the learning of students and colleagues depends on that first step.

# Research on Mathematics Content, Pedagogy and Leadership

Building the expertise of all mathematics teachers requires strategic and targeted PD. According to research, three key elements serve to define the capacity of school leaders of mathematics: (a) content, (b) pedagogy and (c) leadership. The detail and depth of each of these elements varies depending on the role of the lead-learner, hence professional learning must be tailored to support the different roles.

*Mathematical Knowledge for Teaching (MKT)* is a concept developed through the work of Deborah Lowenberg-Ball (2000) who describes how effective teachers must have a composite of knowledge and skills. Unlike mathematicians who apply expert knowledge to further extrapolate/extend the study of mathematics; teachers must be uniquely able to understand how learners develop mathematical concepts. They must also be able to assess their students' current understanding and apply effective pedagogical strategies that move them along this developmental trajectory. This combination of knowledge that links mathematics content and pedagogy is referred to as *pedagogical-content knowledge (PCK*; Schulman, 1986, 1987; Wilson, Shulman, & Richert, 1987). However, there is no agreement on the definition of the concept of *PCK*, and researchers are still trying to develop a better understanding of it; in order to support teacher education and PD of in-service teachers. In the *MKT* model, for example, it is difficult to delineate its subcomponents, and researchers criticize its focus on cognitive aspects of teachers' knowledge (independent of the context in which the teacher teaches).

In their recent paper, Depaepe, Verschaffel, and Kelchtermans (2013) reviewed 60 articles to identify the way *PCK* was conceptualized and (empirically) studied in mathematics education research. They found two perspectives on teachers' *PCK*: *cognitive (*i.e., the knowledge needed for teaching mathematics) and situated cognitive (i.e., dynamic knowledge that is created through the practice of teaching and is recognized in teacher's actions). Depaepe et al. further listed eight components of *PCK*, which include the knowledge of: (1) students' (mis)conceptions and difficulties; (2) instructional strategies; (3) mathematical tasks and cognitive demands; (4) educational ends; (5) curriculum and media; (6) context; (7) content; and (8) pedagogy (p. 15). The authors identified fractions as the popular topic in the *PCK* studies that focused on elementary school teachers, and algebra, along with functions, as popular topics in the *PCK* research at the secondary level.

Depaepe et al. singled out four shortcomings of the strictly cognitive approaches to measuring *PCK*: (a, b) PD may not be adequate as it is separated from both the classroom and the socio-historical contexts; (c) it may not be clear how the components of the *PCK* interact during one's teaching; and (d) affective aspects of teaching may be missing. While the situated cognitive perspective on teachers' *PCK* deals with the stated deficiencies of the purely cognitive model, it has its own deficiencies, such as lack of generalizability because of its overreliance on researching small samples. Depaepe et al. suggested that researchers "clearly state their position in the conceptualization and operationalization of PCK" and "triangulate classroom observations with, for instance… stimulated recall in which teachers can document their choices and justifications" (p. 23).

Likewise, Stein and Nelson (2003) argue the importance of subject matter knowledge in educational leadership. While teachers' knowledge of the subjects they teach is important, little has been done to determine the level and kind of subject matter knowledge that teachers should possess. Similarly, administrators' understanding of subject matter has been neglected in research on educational administration. Stein and Nelson begin by laying out the ideas about leadership and learning, then they present and analyze three cases of instructional leadership situated at different schools and district levels—and examine each for evidence of leadership content knowledge in use. The cases present: (a) an elementary principal doing classroom observations of mathematics instruction and the associated pre- and post-observation conferences with teachers; (b) an associate superintendent leading teachers and parents in the district-level selection of an elementary mathematics curriculum; and (c) a central office team designing district-wide mathematics education reform, (see Table 6).

Stein and Nelson found that as administrative levels increase and functions become broader, leadership content knowledge becomes less fine-grained. Authors suggested that all administrators should have solid mastery of at least one subject (and the learning and teaching of it) and that they develop expertise in other subjects by conducting in-depth explorations of an important but bounded slice of the subject, how it is learned, and how it is taught.

Table 6. Analysis of the three roles and corresponding areas of expert knowledge, Stein and Nelson (2003)

| Elementary principal | Associate Superintendent | Superintendent, his Deputy, and the Director of Mathematics |
|---|---|---|
| Knowledge of mathematics | Less-detailed knowledge of mathematics | Broad, very deep appreciation of:<br>– disciplinary roots of knowledge,<br>– how knowledge is developed and verified,<br>– the role of "schools of thought" in defining what is worthwhile and acceptable |
| Knowledge of:<br>– how children learn mathematics<br>– how teachers can assist their learning | Less specific knowledge of how children construct their mathematics knowledge | An understanding that students must actively construct and interpret knowledge |
| Knowledge of:<br>– how teachers learn to teach mathematics<br>– how others can assist their learning | Lower level of detail about nature of teacher learning | Positioned teachers as mathematics learners |
| Limited knowledge of how to guide learning of teachers within a community. | Knowledge of how:<br>– to set a vision around kind of mathematical thinkers students should become<br>– to use external expertise (the video)<br>– to lead a group process that solicited, respected, and challenged individual points of view. | More robust and well-grounded personal knowledge than knowledge of how to guide learning of mathematics teachers |

A report from a Wellcome Trust study (2013) describes how primary schools in England lead, manage, and teach science and mathematics. The study uses three main sources of data: (a) an online quantitative survey of 209 schools; (b) a set of qualitative telephone-based interviews; and (c) three case study of schools.

Interviews with subject leaders were based on the completion of a role matrix, which listed a range of areas of responsibility that might make up a subject leader's role. Subject leaders were asked to detail whether they are required to undertake each responsibility, and if they are, what this means for them in practice.

The report describes in detail the two main models for science and mathematics teaching in schools, exploring advantages and disadvantages of each, and looking at what would improve the leadership of science and mathematics in schools. It indicates that mathematics lessons are nearly always taught by the usual class teacher and are more likely than science to be taught in ability groupings. Schools are more likely to appoint someone from their senior leadership team to lead mathematics than science. The mathematics leader is likely to coach colleagues, monitor the quality of teaching and learning, monitor achievement, be involved in strategic development, liaise with governors, and hold question and answer sessions for parents (see Table 7).

In an attempt to answer the question: what would improve the way that schools lead science and math? the Wellcome Trust (2013) report recommends the following: subject knowledge of their teaching staff needs to improve and that this is closely linked to a need for providing excellent and subject-related continuous PD, and better resources. Teachers also need time to lead their subjects, as they do not get any additional release time to carry out their leadership role. As well, subject leaders need additional support, whether in the form of extra staff (such as teaching assistants in math) or more subject-specific PD.

Table 7. Comparisons between math and science models used in schools, Wellcome Trust (2013)

| | Mathematics | Science |
|---|---|---|
| Teaching methods | – timetabled every day; usually taught in morning before or after daily literacy lessons<br>– taught in ability groups<br>– greater use of teaching assistants to support small groups work<br>– mathematics leader more likely to teach Year 6 than any other year group | – taught weekly, usually for up to an afternoon at a time<br>– most science leaders evenly distributed over Years 4, 5 and 6 |
| Monitoring learning and achievement | – more formal, more data-driven<br>– More frequent reporting by the math leader-<br>– greater analysis of data<br>– math leaders required to moderate assessment between classes, across year groups and between schools | – moderation not cited as being part of a science leader's remit |
| Monitoring the quality of teaching | – more formal arrangement as lesson observations are timetabled regularly | – monitored via ad hoc observations by science subject leader |
| Working with parents | – math leaders may run workshops about math homework or training sessions for parents to demonstrate calculation techniques | – encourage parents to get involved in informal science at home |
| Professional status | – math leaders are Mathematics Specialist Teachers (MaST)<br>– a teacher with MaST status is:<br>– perceived to be resident math expert within school<br>– teaching colleagues likely to approach them for advice, or with problems relating to math teaching | |
| Budget | – math leaders' budgets likely to be bigger | |
| Extracurricular activities | – more academic and less informal | – more frequent |
| Networking | – more available<br>– math leaders more likely to be members of professional associations | |

## Research on School-based Leadership

Beyond role-specific duties and position titles (such as department chair or grade-level leader), teacher leadership rests with the agency of the teacher to work with the principal. The aims of such work should be to build a learning community, to support teachers, and to determine, implement, or manifest a school-wide vision for instructional practices (Cranston, 2000; Margolis & Huggins, 2012; York-Barr & Duke, 2004).

Fairman and Mackenzie's (2012) assume: (a) that teacher leaders have a means to influence their colleagues' work; and (b) that teacher leaders engage in actions that lead their colleagues to change their practices, however, later they expand the notion of school leadership so that "everyone is a 'school leader'" (2015, p. 80). Fairman and Mackenzie's (2015) consider teacher leadership as an interactive and on-going process, which "evolves just as do the people engaged in it" (p. 81). They question if it is necessary to use the term 'teacher leader', since the term 'teacher' may be encompassing all that should be involved in the job.

Principals play a critical role in supporting effective mathematics instruction. Second only to the influence of teachers, administrators are instrumental in supporting a culture of professional collaboration which prioritizes learning above all else. Cooper et al. (2016), found that:

> …when the principal made room for the voices of teacher leaders and was a member—as opposed to the leader—of the guiding coalition, those teacher leaders effectively drove schoolwide change. Yet, when the principal did not relinquish control or was largely absent, teacher leadership was somewhat stifled and minimized. Fundamentally, then, teacher-leader-driven change must consist of a coalition of teacher leaders with the principal playing nothing more nor less than a supportive role in those particular change efforts. (p. 105).

Cooper et al.'s (2016) findings reveal the changing nature of principals' work. However, principals need some support in order to properly respond to these new expectations. Pollock, Wang and Hauseman (2014) claim that the principals of elementary and secondary schools in Ontario (*N* = 1,423, 52.68% of all principals) reported being overworked to the point of neglecting their own health and well-being (only 2% of them reported balancing life with work), and lacking time for PD. Majority wished to spend less time on management responsibilities (e.g., dealing with student discipline and attendance, and internal school issues), and more time on instructional leadership (e.g., curriculum, instruction, delivery, programming, assessment, and evaluation). Nearly 82% agreed that, "I have been too busy dealing with managerial tasks to give instructional issues the attention they deserve" (p. 29). Principals wished for "More time for professional development so that we can share that with staff" (p. 32-33). The project team suggests that principals "should specifically seek out [leadership skill set] training around four key areas: emotional intelligence/relationship-building; communication skills, knowledge of teaching and learning; and mental health and wellness" (p. 37). Based on Cooper et al.'s (2016) findings, principals may also need specific training regarding aspects of team building and distributed leadership.

Furthermore, Fennel, Kobett and Wray (2013) report that elementary school mathematics leaders are often selected for such positions on the basis of their performance in the classroom. However, research confirms the importance of leadership as a key element of the work and responsibilities of the elementary mathematics leader. In this article, Fennel et al. identify related elements of leadership.

Fennel et al. assert that a major goal, for elementary school math leaders is to help optimize learning for both students and teachers. Nonetheless, "far too many elementary school mathematics leaders have never had the opportunity to formally consider the similarities and differences between child or adolescent learners and adult learners" (p. 174).

For Fennel et al., effective elementary school math leaders understand the needs of their colleagues before they attempt to provide PD, co-plan with their teacher colleagues, or perhaps help establish a professional learning community within their assigned school or schools. Engaging teachers while mentoring or coaching them, requires recognition that each teacher has unique needs.

Fennnel et al. suggest that as elementary school math leaders consider the needs of colleagues as adult learners, they shall take the time to get to know and develop rapport with colleagues. In doing so they may want to note the following considerations:

- Find a way to meet with colleagues individually and informally;
- Find out the content, pedagogical, and interpersonal needs of colleagues;
- Demonstrate respect for colleagues;
- Be mindful of their time, curriculum stressors, personal lives, and so on;
- Consider how to use the problem-centered approach that works best with adult learners;
- Appeal to colleagues by motivating them internally internal motivational (feeling good or successful about their teaching versus an external reward);

- Gauge how colleagues work with children and how their relationships and attention to child needs relate to how they interact with adults;
- Always build reasoning for why something may be relevant for them into PD planning; and
- Provide lead-time and an opportunity to discuss upcoming PD sessions.

In addition, Downton and Sexton (2014) describe a five-year research and professional development project designed and facilitated by staff from Australian Catholic University, and funded by the Catholic Education Office Melbourne. The project aimed to improve mathematics teaching and learning practices in participating schools. Four intakes of schools participated in the project for a two-year period. Data were collected from 25 math leaders across 23 of the schools that participated in the final year of the project. A large focus of the project was to enhance teachers' mathematical knowledge for teaching, through a specialized blend of subject matter knowledge and pedagogical content knowledge related to mathematics learning and teaching.

In writing this article, Downton and Sexton aim to prompt leaders and coordinators to reflect on their beliefs about their role, comparing and contrasting these with the beliefs held by leaders who were involved in the project described.

Findings from the project show that many of the math leaders saw their role in multi-faceted ways. Leaders viewed their role as facilitators of professional learning for their colleague classroom teachers in their schools. They perceived their role as mathematics learning leaders for both teachers and students.

In concluding their study, Downton and Sexton developed a set of questions that could be used by leaders who enact similar roles within their school community to gauge their practice:

> What do you believe is the purpose of your mathematics leadership/coordinator role in your school?
> Compare your belief(s) with those from the leaders from whom we collected data. What is similar about your belief(s)?
> What is different? Why might this be the case?
> What structures are in place at your school that support the enactment of your beliefs about your role? What structures do not enable this to happen?
> How much of your role is dedicated to mathematics teaching and learning?
> What opportunities exist for you to develop this aspect of your leadership role? (p. 5)

## Conclusions

Modern approaches to mathematical instruction in today's classroom may be foreign to some education leaders, who have been taught and trained in more conventional ways. Acquiring necessary pedagogical skills and knowledge to effectively lead teams within their school communities during the new era, requires that they remain open to new professional learning. Education leaders must become lead-learners—educators who are open to new ideas, willing to co-learn, reflect, and change their practices to adapt to and reflect current research findings. For this transformation to happen, leaders as much as all educators need support, which is most often provided through different professional development (PD) opportunities. As Ferguson and Hirsch (2014) note, while PD is significant for creating educators' new knowledge and practices, the quality of PD provided for educators matters significantly. In order for PD to be effective, attention needs to be paid to maintaining collegial relations among educators as they engage in collaborative learning. Not to forget the crucial role community supports play in facilitating such PD opportunities, whether from school administration, school boards and ministries, or the local communities in large.

This literature review confirms that educators in different roles require a role-specific types of subject knowledge, and it confirms the Mathematics Leadership CoP's stand that research, leadership, and teaching go hand-in-hand.

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