Content Leadership in Mathematics Education: A Literature Review 2

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Teacher Leadership

Research in the area of educational leadership has consistently proven over time that school-based leadership teams have a most direct influence on improving the quality of teaching and learning in a school. In a study conducted by Cohron (2009) with teachers from 124 elementary schools in one region of Kentucky, "previous findings (Leithwood & Jantzi, 1999, 2000; Silins & Mulford, 2004) where teacher leadership had no impact on student engagement or student outcomes," were confirmed. Nevertheless, Cohron stressed that "teacher leaders [in formal roles] may provide important support to enhance the instructional capacity of schools with high numbers of at-risk students" (p. 193), in addition to diminishing the pressure on principals, as instructional leaders.

For Lambert and Harris (2003), teacher leadership does not necessarily have to take the form of a formal position, rather "a form of agency where teachers are empowered to lead development work that impacts directly on the quality of teaching and learning" (p. 43). To develop such teacher agency and capacity, leaders who help others mature into leaders, are especially needed (Fullan, 2005; Harris, 2005). Calderone, Kent, and Green (2018) precisely identify qualities of teacher leaders:

> *Teacher leaders have the same attributes of good teachers: open-mindedness, flexibility, confidence, and experience in their fields. They also …:*
>
> *Actively listen; Facilitate meetings; Decide on a course of action; Monitor progress; Have expertise in their field; Encourage success from their colleagues; Have expertise in teaching and learning; Facilitate communities of learning through organization-wide processes; Confront barriers in the school's culture and structures; Translate ideas into sustainable systems of action; Strive for authenticity in their teaching, learning, and assessment practices. (Calderone, Kent, & Green, 2018, p. 396)*

Masters' (2010) *Teaching and Learning School Improvement Framework* identifies leadership practices, which appear to be most directly related to school-wide improvements in teaching and learning. Crucial in this model is for every school to, "build a professional team of highly able teachers including teachers who take an active leadership role beyond the classroom" (p. 9). This framework underscores the importance of a school culture where all are responsible for student learning and success, where educators jointly analyze samples of student work; co-plan, co-teach, and review efficiency of their teaching; and celebrate and encourage professional learning.

> *Effective leaders create cultures of high expectations, provide clarity about what teachers are to teach and students are to learn, establish strong professional communities and lead ongoing efforts to improve teaching practices. (Masters, 2010, p. i.).*

[1] This literature review was conducted by the Mathematics Leadership Community of Practice research team.

A similar idea was adopted in the Ontario's Renewed Math Strategy (RMS), which required that every elementary school has math lead teachers, "whose responsibility is to deepen their math knowledge through professional learning, to apply this learning in the classroom and to share strategies for learning with other teachers in their school" (Ministry of Education, 2016). The novelty of the RMS, compared to frameworks such as Masters' (2010), was in its strong focus on leadership in mathematics education (Stein & Nelson, 2003), which prompted us to investigate a largely unexplored idea of content leadership. In order to better understand what mathematics content leadership entails, we conducted a literature review from which we created a model of effective mathematics leader as one who is: (a) aware of organizational change goals, (b) knowledgeable of mathematics content, (c) informed about pedagogical practices, (d) connected with education community, (e) embedded in educational context, and (f) experienced with action research (see Figure 1, Martinovic, Horn-Olivito, & ElKord, 2017). The present document provides insights from additional and more recently published resources.[2]

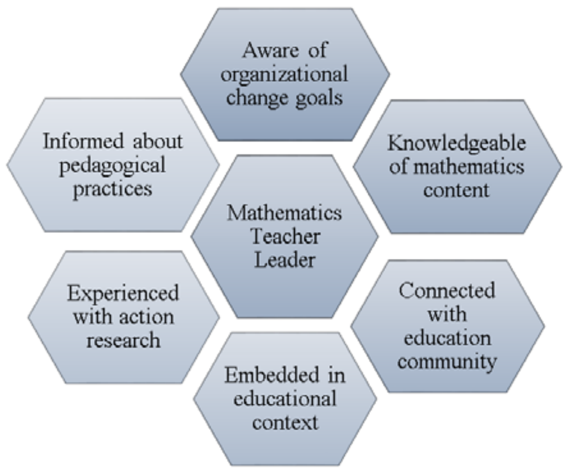

Figure 1. An emerging model of mathematics teacher leadership--working with adults to benefit students (Martinovic, Horn-Olivito, & ElKord, 2017).

In the conclusion of this document, we provide a revised model of the mathematics teacher leadership, based on the findings from our literature reviews from 2017 and 2018.

Selecting and Preparing Teacher Leaders

Based on the studies confirming that teacher leadership positively impacts teacher practice and, by extension—student outcomes, some researchers embarked to explore aspects of teacher leaders' selection and professional programs for their preparation.

Welch and Hodge (2018) conducted a case study in one Georgia school district to develop leadership competency models appropriate for the district's context, utilizing the previously developed Welch-Bussey *Framework of the Essential Dispositions of School Leaders* (Bussey & Welch, 2014). Many of the district leaders and principals had already participated in a training program prior to the onset of the case study—a camp and leadership summit—organized by Georgia Leadership Institute for School Improvement (GLISI), which also served to establish trust between practitioners and GLISI researchers. After that, in 2013, the district leaders partnered with GLISI to develop the *Core Elements of Leadership Excellence* for their district. A core group of 10-11 leaders at the school and district level composed a Design Team, to engage in identifying the skills, competencies and dispositions required to be a successful leader in the district. GLISI's Director of Strategic Consulting was responsible for facilitating the team's work. Design Team members shared their opinions

[2] In other documents we present results from our ongoing case studies.

about important characteristics of the district's leaders. In addition, GLISI brought seminal research, tools, and resources to the table to help challenge the thinking of the group. Meetings began with conversations around the team's own understanding of the leadership skills, qualities, and dispositions required for success, and were checked against the research and policy documents, to ultimately arrive at a framework that was reflective of the Design Team members' collective thinking. As the work of the Design Team yielded the district's Leadership Competency Model, GLISI supported the District in developing a selection process to identify aspiring leader candidates and the curriculum for the aspiring leaders' program.

Throughout and following the program's implementation, Welch and Hodge collected and analyzed data from interviews and focus groups with the Design Team and the Aspiring Leaders Alumni, as well as from source documents and other program evaluation data. They found that GLISI's support and prior research on leadership dispositions associated with effective school leadership, were critical to the development and successful implementation of the county's Leadership Competency Model. The district's Leadership Competency Model created a critical foundation for building qualified leaders in the school system. In particular, the development and reinforcement of the competency model has led to "(1) increased clarity among aspiring leaders about the expectations for leadership within their system; (2) increased shared understanding among school leaders and district administrators about the core elements of leadership required for success within their system; (3) increased uniformity of practice in the identification and selection of candidates into the Aspiring Leader Program; and (4) supported improvements in the development of aspiring school leaders" (p. 366).

Welch and Hodge's findings raised questions about the importance of screening for a core set of qualities or dispositions as part of the selection process into an aspiring leaders program; key dispositions that they urge such programs to consider include *respect*, *care*, *commitment*, *integrity*, *trust*, a *passion for students* and a *belief that all children are capable of learning* (the last two being necessary for acceptance into a program or a leadership role). They emphasize that a professional learning experience that focuses only on leader standards miss an opportunity to cultivate critical dispositions for future success. Welch and Hodge also suggest that district office staff should on a regular basis engage in conversations with leadership program personnel and the teacher leaders, as the richness of conversation during the Design Team phase of their project's partnership was as valuable to district office staff as the leadership competency framework that ultimately emerged. Finally, they emphasize the importance of keeping a district's leadership competency framework malleable and flexible over time, as demands on school leaders are ever changing.

Research on the Impacts of Teacher Leadership

Examining the impacts of teacher leaders' work on school and student outcomes, a nationwide Australian research project, *Building an Evidence Base for National Best Practice in Mathematics Education (BPME),* aimed to identify school practices and policies that showed growth and/or improvement in their numeracy results. Success was defined in this project as growth in students' numeracy scores from grade 3 to grade 5, or grade 7 to grade 9, in National Assessment Program—Literacy and Numeracy (NAPLAN)[3].

Using data collected in the BPME project, Muir, Livy, Herbert, and Callingham (2018) conducted a small-scale case study that focused on three P-12 schools, which demonstrated substantial growth in NAPLAN scores (i.e., "over one standard deviation above the mean growth for like schools or schools with the same starting score," p. 302). They used Masters' (2010) *Teaching and Learning School Improvement Framework* in an attempt to understand how school working conditions transfer into student outcomes. Masters'

[3] Similar to the EQAO standardized testing in Ontario, NAPLAN testing has been used since 2008 across Australia to assess students in grades 3, 5, 7, and 9 in literacy and numeracy. The numeracy test includes topics such as numbers, geometry, algebra, functions and patterns, measurement, chance and data.

framework is based on the eight domains, each of which identifies one important aspect of a school's day-to-day practice. The domains require that:

1. The staff follow an explicit and clear school improvement agenda,
2. The school and teachers organize their actions around discussion and analysis of data,
3. The school culture promotes learning for all students and continual improvement of teaching,
4. The school resources (e.g., staff time, expertise, funds, facilities, materials) are used purposefully,
5. The school has built an expert teaching team that leads and models professional learning in the school,
6. The school has a well-aligned systematic curriculum development and delivery plan,
7. The school prioritizes differentiated classroom learning, and
8. The staff are committed to exercising research-informed effective teaching practices.

Muir et al. (2018) found a number of influential characteristics that aided in the successful improvement of NAPLAN numeracy scores; most impactful of all were those related to school-wide approaches to the teaching and learning of numeracy. Such approaches included: (a) intentional policies and programs to support numeracy growth, (b) strong school and numeracy leadership, and (c) the use of data from several sources to inform future teaching and learning directions.

In addition, it was evident that one of the contributing factors to school success was that teachers were not being instructed to teach to the test. On the contrary, the three schools' principals and teacher leaders deliberately toned down the instructional focus on increasing test scores (NAPLAN scores) within their schools and actively worked to alleviate potential stress about testing on staff and students. This eventually had a positive influence on the schools' improvement agenda, including raising the standards of mathematics teaching and learning.

All three successful schools "highly valued" credentials for teaching mathematics (p. 307). The principals of these P-12 schools, preferred that the new mathematics teachers be certified high school mathematics teachers, who are comfortable with teaching elementary school content. All high school mathematics teachers were expected to teach only mathematics, but also teach Grade 7 students, while supporting PD of other staff members. Muir et al. concluded that there could be several factors which enable schools to demonstrate growth in numeracy results including some that are contextual and influenced by student cohorts, school culture, leadership and personnel.

With these factors in mind, while implementing a school improvement agenda and plans for enhancing mathematics teaching and learning in schools, research suggests that it is essential to effectively equip mathematics leaders with the tools they need to best support mathematics teachers. Studies recommend paying close attention to the methods and tools employed for evaluating and assessing teachers' professional knowledge. In the next sections, we present a brief review of recent literature on teacher leadership and teacher PD that conceptualize mathematics teachers' content knowledge and recommend certain PD models.

Teachers' Identity and Professional Knowledge – An Ongoing Process

In a recent publication, Schauer (2018) explores themes and moments of teachers' critical consciousness and declares that the 1970's teacher preparation programs were based on the assumption that teacher identity and knowledge were primarily formed within the context of the four-year university teacher preparation programs. The 1980's however, witnessed a shift in this perspective, as teachers were more seen as "reflective practitioners" (p. 2). As such, they were expected to reflect on their day-to-day work in the field and to engage with colleagues in continual improvement of their practice. Schauer stresses that "This shift from 'I am a teacher' to 'I am becoming a teacher' allows teachers to think of themselves and their professional identities as a continually evolving process. Learning to teach becomes 'a process and not an event' (Cochran-Smith, 2012, p. 109)" (p. 2). Schauer further emphasizes that this shift in perspective is particularly important in the North American urban environments, where predominantly White and female teachers constantly need more time and support to engage in reflective practices to help them better understand and reach their diverse student populations.

Following this shift in perspective that occurred during the 1980s regarding teachers' professional learning and knowledge, mathematics education researchers started to increasingly explore teachers' content and pedagogical knowledge (Neubrand, 2018). Apparently, it became clear that teaching mathematics requires specific mastery, although, "if the content really plays a decisive role depends on the viewpoint of the researcher" (p. 603).

Conceptualization of Teachers' Professional Knowledge

Lee Shulman's address as a President of the AERA (Shulman, 1986) was a revolutionary call for a deeper understanding of teacher knowledge, so that its contextual, content-specific, and developmental aspects (from the perspective of students) are clarified. Shulman "proposed to distinguish three fundamental dimensions of teacher knowledge: content knowledge (CK), pedagogical content knowledge (PCK), and generic pedagogical knowledge (PK)" (Neubrand, 2018, p. 602). Since then, Shulman identified some weaknesses of the original PCK framework—it presented teachers' knowledge as emotion- and affect-free, and teachers as thinkers, problem solvers, and decision makers, forgetting the aspects of the skilled performance. It also lacked connection to the broader social contexts, and teachers' vision of what a good and just society ought to look like (Center for Engaged Learning, 2014). Consequently, various research groups attempted to further the notion of PCK and during the last two decades, several frameworks have also been proposed to capture the essence of the knowledge needed for teaching mathematics (e.g., Adler & Davis, 2006; Ball, Thames, & Phelps, 2008; Davis & Renert, 2013).

In an effort to investigate the validity of such frameworks, different approaches have been carried, ranging from (a) exploring connotations of teacher knowledge, to (b) the quality of instruction and/or student learning and outcomes (e.g., Hill, Rowan, & Ball, 2005; Kersting, Givvin, Thompson, Santagata, & Stigler, 2012; Shecthman, Haertel, Roschelle, Knudsen, & Singleton, 2013), to (c) studying the multi-dimensionality of teacher knowledge (e.g., Charalambous, McGinn, Hill, & Chin, 2014; Kleickmann, Richter, Kunter, Elsner, Besser, Krauss, & Baumert, 2013). In addition, various research teams have expanded on these conceptualizations and developed their own tools and instruments to measure and assess teachers' professional knowledge.

Research on Professional Knowledge for Teaching Mathematics

Similar to many other researchers, Ball, Thames, and Phelps (2008) closely studied Shulman's model of teachers' professional knowledge. They reproduced an articulation of Shulman's Major Categories of Teacher Knowledge (see Figure 2), and conducted a study that resulted in defining an additional domain, namely, Mathematical Knowledge for Teaching (MKT). Ball et al.'s (2008) theorizing of the MKT described how effective mathematics teachers must have a composite of knowledge and skills. Unlike mathematicians, mathematics teachers must be uniquely able to understand how learners develop mathematical concepts and be able to assess their students' current understanding, in order to apply effective pedagogical strategies.

- General pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter;
- Knowledge of learners and their characteristics;
- Knowledge of educational contexts, ranging from workings of the group or classroom, the governance and financing of school districts, to the character of communities and cultures;
- Knowledge of educational ends, purposes, and values, and their philosophical and historical grounds;
- Content knowledge;
- Curriculum knowledge, with particular grasp of the materials and programs that serve as "tools of the trade" for teachers;
- Pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding.

Figure 2. Shulman's Major Categories of Teacher Knowledge (Shulman, 1987, p. 8; as cited in Ball, Thames, & Phelps, 2008, p. 391)

Following this, several researchers further explored the validity of Ball et al.'s (2008), MKT construct. For example, Scheiner, Montes, Godino, Carrillo, & Pino-Fan, (2017) summarized a distinction between Shulman's PCK and Ball et al.'s MKT in the following way:

> *Shulman's notion of PCK puts emphasis on a kind of knowledge distinctive to teachers (and not to disciplinary scholars) and Ball and her colleagues' notion of specialized content knowledge puts emphasis on a kind of knowledge distinctive to mathematics teachers (and not to teachers of other subjects). (Scheiner et al., 2017, pp. 6-7)*

Similarly, Charalambous (2016) conducted a study focusing on four *teaching practices*—actions in which teachers engage as they teach mathematics, such as: (a) *providing explanations*—"presenting ideas, …responding to students' questions, or …scaffolding struggling students" (p. 222) and *evaluating students' explanations*; (b) *selecting and using representations* (i.e., which representations may be appropriate for the learning goal and the students; when and how to use them); (c) *analyzing students' errors, misconceptions, and non-conventional solutions*, so that they can "be fruitful in reorganizing the understanding of learners" (Shulman, 1986, p. 10).; and (d) *selecting tasks*.

Results from Charalambous' study suggest that although knowledge of mathematics is important, it might not suffice for successfully carrying out the work of teaching it. Using a sample of 644 Greek-Cypriots pre-service and in-service elementary school teachers and university students (studying for at least four semesters in departments of mathematics, computer science, and architecture), Charalambous found:

- significant differences between in-service and pre-service teachers (and between teachers and undergraduate students with strong background in mathematics), with respect to *providing explanations* and *using representations*;
- no significant difference between in-service and pre-service teachers on *analyzing student errors, misconceptions, and non-conventional solutions*, and
- no significant difference between in-service teachers and undergraduate students on *selecting tasks*.

The study's results grant credibility to the idea that teacher knowledge should not be regarded as a static entity, but a constantly evolving one, throughout teacher preparation and career. Consequently, Charalambous emphasizes that research on PCK, as well as PD sessions, should focus not only on knowledge that teachers (ought to) have, but also on teaching practices, of which the two—providing explanations and using representations—are the most prominent and distinctive. He thus advises that courses and PD programs should be designed to create engaging opportunities for pre- and in-service teachers to expand and refine their knowledge for teaching. In addition, the study underscored the importance of using a mixed-methods design in validating teachers' professional knowledge frameworks and approaches; recommending using alternative approaches to multiple choice survey items, to tap into teachers' knowledge, including open-ended questionnaires and interviews, or observing and responding to different scenarios portrayed in videotaped or simulated teaching episodes.

In the same realm, and in an effort to explore issues specific to mathematics teaching, Neubrand (2018) presents three influential, globally recognized projects that were respectively conducted in Michigan, Germany, and internationally. Drawing on these projects, Neubrand sketches basic ideas of selected theoretical approaches, their goals, their fields of application, and their methods that address the components of the knowledge that teachers need for teaching mathematics with professional consciousness.

In alignment with Schauer's (2018) emphasis on the importance of teachers' reflective practices to make meaning of urban contexts, that we referred to earlier, Neubrand (2018) explains that "Shulman's central aspect of CK is largely also determined by the pedagogical needs, and conversely his PCK conceptualization draws heavily from content driven considerations and reflections" (p. 603). Neubrand emphasizes that "[t]he contextual factors which are …typical for 'a profession'…bring along some limitations one should bear in mind, especially when models of knowledge are going to be applied in the field, and when they are used as a basis for decisions" (p. 609). He notes *the gap between knowing and acting, the cultural context,* and *the affective component,* as examples of such limitations. These limitations and Neubrand's suggestions are explained in the following section.

Evaluating PCK of Mathematics Teachers – to develop effective PD programs

The complexity of defining and assessing knowledge for teaching mathematics is evident in different conceptualizations and disagreements among education scholars. For example, Scheiner et al. (2017) criticize approaches that compare mathematics teachers' knowledge with that of mathematicians or teachers of other subjects. They claim that it is more useful to look within the domain of teaching mathematics for contextual factors that shape actions of mathematics teachers: "[mathematics teacher knowledge] is less about static dispositions or traits differentiable from those of other professions and more about the complex dynamics of the usage and function of knowledge in context" (p. 10). Scheiner et al. emphasize that mathematics teachers' knowledge goes beyond the skill of unpacking mathematics for their students, it also involves unpacking student understanding of mathematics to make this understanding visible. In that way teaching mathematics is not only top-down, but also bottom-up, as it needs to situate knowledge in the knower and make it culturally appropriate. In addition, Scheiner et al. see "teacher knowledge as an organic whole" (p. 12), which aligns with Shulman's notion that PCK is an amalgam of different knowledges. Such knowledge is both *additive* (as components add to each other) and *transformative* (as components blend and transform each other). In conclusion, Scheiner et al. advise to abandon the approach of fixing mathematics education by "fixing teachers' lack of knowledge" (p. 16). Instead, teacher education and PD programs should consider teachers' knowledge as constantly developing and growing, and help teachers incorporate insights from various disciplines, such as mathematics, education, and psychology.

According to Neubrand (2018), models for evaluating teachers' professional knowledge need to pay attention to closing the evident gap between the knowledge per se and the need for acting in the classroom. In a nutshell, the PCK cannot be assessed nor developed without a classroom context. Cognitive tests can only provide a rough estimate of teachers' PCK, but the full picture would require observing them in action (in the real classrooms).

Neubrand concludes that the idea of theory-building is an often-neglected aspect of teachers' professional knowledge, which in turn lacks in their teaching. According to Weiss and Herbst (2015), "[t]he theory building disposition places a high value on the organization of a body of mathematical work… [it mainly focuses] on illuminating the relationships among existing pieces of mathematical knowledge" (p. 207). This disposition extends the prevalent view among mathematics teachers that the purpose of schooling is to enable students to problem solve. Based on the study with about 20 experienced teachers, who all taught Geometry for at least two years, Weiss and Herbst concluded that the teachers' prevalent

> *uncertainty about the axioms underpinning the course they teach may be largely a function of the confusing state of the curriculum materials they use, but it is also indicative of the low value they place on holding students (and themselves) accountable for the theory building disposition. (p. 218)*

One example of a theory-building would be to teach students not only the separate facts, such as that the corresponding and alternate angles on a transversal are congruent, but discussing with them that these statements are mathematically equivalent (meaning that if anyone is taken as true, the others would follow from it as consequences). Similar disconnects we saw in many textbooks where, for example, the simple interest formula is taught as I = Prt, while the principal is taught as $P = \frac{I}{rt}$, and rate of change as $r = \frac{I}{Pt}$. All these formulas could be algebraically developed one from another, and teaching them separately forces students to learn them by heart, rather than understanding where they come from and using algebra to infer the needed formula.

Another, affective, constituent of teacher knowledge that Neubrand (2018) underscores, consists of two key components, namely "being interested in the personal and intellectual development of the learners, and being informed about the foundational elements that rule the scholarly domain of teaching" (p. 609). In addition, "the mechanisms of doing mathematics, i.e., the 'practices, sensibilities, and ways of thinking of mathematics' (Bass, 2017, p. 229), should also be part of the professional knowledge of teachers" (p. 609).

Neubrand argues that creating a holistic knowledge of mathematics is a goal of teaching mathematics, and reaching that goal requires "more than categories rooted in instructional design only, but needs to be informed about central mathematical practices" (p. 610). He thus points out that "a significant part of teachers' professional knowledge should be devoted to building up specific mathematical knowledge, not sheer instrumental knowledge, but in direct relation to the educational issues a teacher is faced with during teaching" (p. 610).

Teachers' PCK Assessment Models

In an effort to guide mathematics leaders and teachers' PD facilitators in evaluating teachers' professional needs and developing effective PD plans and programs, researchers have developed instruments for a range of mathematical disciplines (e.g., geometry, algebra) over time. In this section, we introduce a number of recent publications that offer some assessment tools and recommendations for mathematics teachers' PD.

Assessment tools -"probes" around specific topics. In their longitudinal research with four cohorts of in-service secondary mathematics teachers, Martinovic and Manizade (2018) focused on exploring and designing assessment instruments for measuring teachers' mathematical knowledge for teaching geometry. Their assessment of teachers' PCK, or related constructs of teachers' knowledge, goes beyond inspecting teachers' ability to solve given mathematical problems. They suggest that it would be more effective to select a small number of commonly taught mathematics topics and to then develop instruments that identify a teacher's competency level in those areas (Manizade & Martinovic, 2016; Martinovic & Manizade, 2018), rather than probing for a broad range of topics.

Martinovic and Manizade (2018) designed assessment instruments that can be used for measuring teachers' PCK in geometry, and specifically in teaching the formula for finding a trapezoid area. For their study, Martinovic and Manizade designed a probe that includes: (a) a set of students' hypothetical exemplars, for teachers to analyze; and (b) corresponding rubrics for evaluating the teachers' qualitative responses. Using a Grounded Theory approach and multiple sources of data, they tested and adjusted the rubrics over the span of the study to develop simple but detailed enough instruments to be used in conjunction with the Trapezoid PCK instrument, developed within an earlier study (Manizade, 2006). Their aim was to develop rubrics for evaluating teachers' responses that offer insight into the level of teachers' PCK, which could ultimately lead to a differentiated PD. Diagrams that they used for visualizing teachers' PCK profiles, (a) clearly show areas of strength as well as areas where PD is needed, such as asking diagnostic questions and understanding student challenges and conceptions, or (b) that the teacher lacks knowledge of geometry, or (c) has trouble facilitating further student thinking and helping them to extend their approach. These diagrams are also appropriate for self-assessment as they could be easily interpreted by a person untrained in using the PCK instrument and rubrics (see Figure 3). The example in Figure 3 belongs to a fictional teacher who seems to have solid knowledge of Geometry and knowledge of student's challenges and conceptions (Level 3 on the scale 0-4), but may need help in developing the other three areas.

Figure 3. Visualization of teacher's competency in five domains that relate to teaching Geometry.

Manizade and Martinovic (2016) proposed to provide such content-specific, short, interactive, student exemplars-based instruments online, so that teachers could access them and do a self-evaluation individually or in groups.

Professional Learning Communities. According to Warwas and Helm (2018), recent literature reviews and meta-analyses indicate that high-performing schools foster various forms of teacher cooperation and ongoing PD, such as professional learning communities (PLCs). Such PLCs take different forms, from small interdisciplinary teams to teams that consist of the entire school staff or district units. Warwas and Helm especially focus on the three core dimensions that describe the way collective professional practice is orchestrated within a PLC, namely *collaborative development*, *normative agreement*, and *supportive infrastructure*.

According to Warwas and Helm, *collaborative development* denotes the behavioural dimension of a PLC, where members collaborate to improve pedagogical practices by (a) exchanging instructional materials, (b) collectively designing and implementing instructional units, thereby establishing a growing set of shared instructional practices and methods, and (c) continually conducting collective inquiries. Besides engaging in reflective dialogue about instructional issues, such professional learning activities may include mutual visits and consultations, peer coaching, and joint evaluation of instructional processes and outcomes.

On the other hand, *normative agreement* describes the ideational dimension of a PLC, where educators' efforts serve shared purposes, pedagogical values and goals, and personal beliefs about effective teaching and learning. And *supportive infrastructure* depicts the structural dimension of a PLC, where a wide range of aspects determine an organization's capacity for PLC, two of which are particularly important: (a) smooth and coordinated routines (e.g., clear rules, actions and duties; access to relevant information; efficient communication), and (b) sufficient time and meeting space available for the teams.

For their study conducted among vocational schools in Germany[4], Warwas and Holm investigated how distinct configurations of PLCs in school departments relate to instructional quality in the classrooms. They first explored departmental PLC profiles, using dimensional scores on collaborative development, normative agreement, and supportive infrastructure as classification criteria. They did this in order to examine if, and to what extent, variations in instructional quality are attributable to teachers' membership in distinct PLC profiles. They inquired whether teachers who belong to (comparatively) strong PLCs, indicated by high scores in all three dimensions, do indeed provide students with better opportunities to acquire knowledge and skills. Warwas and Holm (a) first surveyed teachers to assess the degree to which their departments follow core PLC dimensions; (b) and then students, to find the degree to which teachers demonstrate effective classroom management, individual learning support, and application-oriented teaching, and the frequency in which teachers employ action- and application-oriented methods such as problem-solving.

Based on their findings, Warwas and Holm conclude that teachers from advanced PLCs do indeed provide better opportunities for students to acquire knowledge and skills by creating more authentic, application-oriented learning environments. They contend that distinguishing departmental collective professional practice within a PLC—based on behavioural, ideational, and structural dimensions—can offer useful hints for educational research and practice. Warwas and Holm's data point out that having a supportive environment does not always result in teacher engagement or even agreement. However, teachers' beliefs of their work-related efficacy was predictive of their participation in PLCs that were strong in all three domains. These results align with findings from Vescio, Ross, and Adams (2008), implying that teachers need to be convinced to collaborate and to change a likely engrained mind-set rooted in the culture of individualism, autonomy, and independence.

Without an essential amount of openness to collaborate, every effort pushing teachers towards collaboration may become lost in a culture of contrived collegiality.
(Vescio, Ross, & Adams, 2008. p. 36)

Warwas and Helm (2018) suggest that education leaders systematically describe and critically evaluate collective professional practice within departments, in light of desirable features of work in a PLC. Such departmental profile analysis can also help teachers in identifying potential deficiencies in their respective departments and to thus plan targeted measures for improvement.

Research on Developing Effective PD Programs

While it is crucial to evaluate teachers' professional knowledge before and during the development of PD programs, research suggests that designing the programs around the perspectives of the teachers themselves may be as crucial.

Designing heuristic teachers' PD programs. Based on an extensive literature review, Choi and Walker (2018) identified key school reform features that shape teachers' engagement in the era of change, and suggested a framework for designing PD programs that address reform-specific needs. Then, they explored the utility of this framework through an in-depth case study that addressed implementation of, and perceptions about, eight reforms in two Hong Kong schools.

[4] In Germany, vocational schools comprise of both secondary and post-secondary institutions which offer specialization in distinct sectors, e.g., technical-industrial, social, commercial, or technological.

The heuristic model of Choi and Walker is a matrix (see Table 1) addressing the two aspects that appeared as most prevalent in the reform-related research literature, namely ambiguity and conflict generation. By ambiguity and conflict they refer to tension that some educational reforms incur between newly promoted changes and teachers' beliefs. That is, for a reform to be successful, the policy makers and PD providers need to ensure "that [adequate] knowledge (know-what) and skills (know-how) are provided" (p. 74) to teachers. The matrix they proposed incorporates four types of PD programming; (a) informative, (b) experiential, (c) negotiatory, and (d) experiential and negotiatory, to be matched with educational policies of different levels of ambiguity and conflict.

*Informative PD*s are appropriate for reforms with low ambiguity and conflict. In such cases, the PD can consist of talks and information-sharing seminars for a wide range of audiences. These unilaterally conducted, transmissive, seminars mainly aim to inform and help participants understand the required changes, and equip them with the basic, straightforward skills necessary for the reform implementation.

The highly ambiguous reforms require e*xperiential PDs*, in the form of workshops, mentoring, and peer support groups. This form of PD involves a considerable degree of interaction and collaboration with experts or peers, aiming to provide ample opportunities for teachers to apply and reflect on a promoted teaching practice.

The reforms that are in conflict with teachers' beliefs or identities would require *negotiatory PD* in order to provide suitable opportunities to address the conflicts involved. Such PD programs must focus on communication, persuasion, and negotiation, and may include open discussion and consultation during which teachers and PD leaders discuss the agenda for narrowing the gap between required changes and teachers' beliefs and identities. The outcome may be two-folded, where teachers may accept the reform or the policy makers may revise the reform to adjust it based on the feedback they receive from the public.

Experiential and negotiatory PD however, would best work for educational reforms that appear as highly ambiguous and conflicting. Such reforms require multidimensional changes, including the transformation of beliefs, teaching and learning materials, and pedagogical practice. Choi and Walker give an example of one such mathematics education reform in the US, called The Qualitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) project. This national educational reform project called for middle school mathematics instruction, in which all students can learn mathematical concepts and deeply understand mathematical ideas, while demonstrating proficiency in reasoning and problem solving. Such an approach required a shift from results-oriented pedagogy to development-focused teaching, while emphasizing reasoning and communication skills. Although starting with some hiccups, this reform was successful at providing substantial support to teachers; through (a) 20 training and discussion sessions that helped teachers change their perspective and understand the reform better, and (b) mentoring teachers in their schools, throughout the three years of the project. Participating teachers received release time, honorariums for attendance, and credits for graduate studies.

Table 1. Matrix for customizing a teachers' PD program to the characteristics of a reform (Choi & Walker, 2018).

	Low Conflict	**High Conflict**
Low Ambiguity	Informative PD	Negotiatory PD
High Ambiguity	High Experiential PD	Experiential and Negotiatory PD

To examine the utility of this framework, Choi and Walker conducted a multiple case study that explored the relationships between the perceived reform characteristics, the frameworks' PD types, and teachers' perceptions of eight reforms across two schools. For the analysis they used data from individual and focus-group interviews with 24 teachers of different ranks at the two schools, as well as the records of the PD programs provided to the schools by the Hong Kong Education Bureau (EDB) and other institutions over 14 years, from1999 until 2013, interviews with teacher leaders, and school documents.

The findings of Choi and Walker's study emphasize that customizing PD provision to perceived reform characteristics should be an on-going process throughout the reform, as other, unexpected, issues may arise. They thus recommend that teacher leaders begin reform implementations with an initial assessment of degrees of ambiguity and conflict surrounding a policy, through surveys or focus group interviews. This would provide insights into what type of PD should be provided, in addition to structural information on the potential problems and legitimacy of the policy itself. They suggest that it may be necessary in some cases to explore the need for providing school-specific, or even department-specific PD. They argue that factors such as the schools' historical relationship with the initiative, and teachers' background and competencies existent within a school can come together to create different needs across schools, and sometimes even within a same school.

Choi and Walker also recommend bringing both policy makers and practitioners together to analyze and categorize reform from teachers' experiential perspectives, while taking into consideration suggestions from reform initiators and leaders, in order to identify effective PD provision. They believe that this could contribute to teachers' positive perceptions of and a productive experience with reforms and PD programs.

Conclusions

While several factors contribute to improvements in teaching and learning, some are contextual and are influenced by student cohorts, school culture, leadership, and personnel. School-wide improvements require a school culture where collaboration, trust, and professional learning are fostered. According to research, school-based leadership has a most direct influence on improving the quality of teaching and learning in schools, particularly those with high numbers of at-risk students. While teacher leadership can take the form of formal positions, it can also be manifested in the form of teachers empowered to lead teaching and learning development. Lead teachers (see Figure 4) are expected to be well aware and focused on change goals, while carrying the responsibility of deepening their content knowledge and advancing their pedagogical practices through professional learning, and to apply this learning in the classroom while sharing strategies for learning with other teachers in their school and a wider education community.

Literature also suggests that in addition to formal expectations, effective teacher leaders need to possess attributes of open-mindedness, respect, care, commitment, integrity, trust, flexibility, encouraging and inspirational attitudes, confidence to plan and carry a course of action, engaging facilitation skills, authenticity, experience in their fields, a passion for students, and a belief that all children are capable of learning.

For teacher leaders to effectively lead teaching and learning improvements, they need to view teachers' professional identity and knowledge, in its three fundamental dimensions: content knowledge (CK), pedagogical content knowledge (PCK), and generic pedagogical knowledge (PK), as a continually evolving process. They also need to realize that although knowledge of mathematics is important, teaching mathematics is not only top-down, but also bottom-up. Mathematics teachers need to be able to understand how learners develop mathematical concepts and be able to assess their students' understanding, and apply effective pedagogical strategies. This implies gearing PD towards theory-building and teaching practices, particularly providing explanations and using representations in teaching, to create engaging opportunities for teachers to expand and refine their knowledge for teaching.

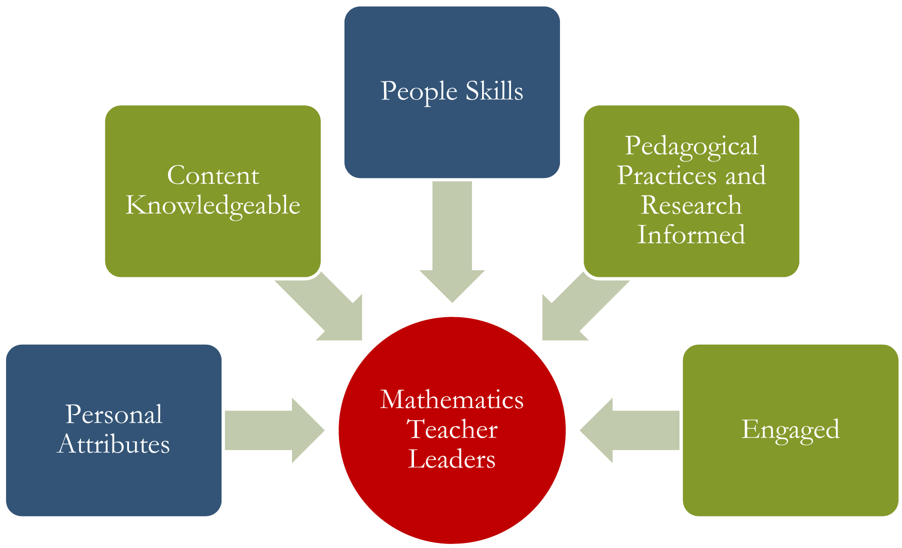

Figure 4. Characteristics of mathematics teacher-leaders.

Leading effective PD needs to begin with an initial assessment of mathematics teachers' PCK and the degrees of ambiguity and conflict surrounding a policy or change goals, thus customizing PD provision as an on-going process. Not to forget the importance of engaging policy makers and teachers in conversations to effectively contribute to teachers' productive experience with reforms and PD programs.

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