Eastwood Whole School Inquiry on Concreteness Fading[1]

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Background

How do students learn mathematical concepts, strategies, reasoning, and procedures? How do students use Mathematics to make sense of new concepts and ideas? How do educators best support student learning of mathematics? We believe that students need to learn mathematics and they need mathematics to learn, but deeply understating how this learning occurs over time and how to best support it becomes a vast area for debate and study.

In the fall of 2016, Eastwood Public School, an urban elementary school of 270 students within the GECDSB learned about Concreteness Fading—a process in which the a mathematical construct is faded from concrete materials, including physical, virtual, and pictorial objects, to abstract representations (Fyfe, McNeil, Son, & Goldstone, 2014). A growing body of research indicates that students who use concrete materials for mathematics learning generally outperform students who do not (Moyer, 2001). Being supported in much of the literature, the process of Concreteness Fading holds much promise as a pedagogical approach, but a practical implementation of the process was considerably lacking.

Eastwood Public School set out to develop an understanding of how concreteness fading could inform planning of tasks, lessons, and units, both within and across grades. The teachers who were part of this inquiry were seeking to put research into practice.

The inquiry team included all educators in the school. There was a core team that consisted of teachers and the Principal and Vice Principal, who led the inquiry. Other teachers supported the inquiry by reading research, testing lessons and units, providing feedback, and analyzing student data.

Early on, the team developed their research question: *How is the pedagogical approach of Concreteness Fading applied in the elementary classroom?* This report summarizes their findings.

This report includes a captions of *key learnings*—pivotal moments in the inquiry which served to deepen and strengthen the work. These captions include reflections or teacher quotes, taken from meetings or conversations.

What does it mean to be good at math?

Several years ago, when the school was part of intensive learning as an OFIP school, the staff closely examined the mathematics curriculum as well as many resources provided by the Ontario Ministry of Education. As a result, educators at Eastwood have a good understanding what the Ontario Mathematics Curriculum asks students to demonstrate. They have well-developed ideas about mathematics content and pedagogy for the grades they taught. When the construct of mathematical proficiency was formally adopted by the GECDSB in 2014, it helped the staff clearly articulate what they had grown to understand and infuse in their school culture (National Research Council, 2001).

The learning culture of the school was predicated on the idea that all children can learn mathematics to the highest degree and that their goal for each child is to achieve strategic competence, adaptive reasoning,

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conceptual understanding, and productive disposition. The question remaining was how to best support their students in attaining this goal.

Mathematics manipulatives have been in existence in many classrooms for decades. In the Ontario context, concrete materials are familiar. Building consistent and effective applications is part of the current work happening in many school boards. According to data gathered from the GECDSB Math Task Force Report (2016) concrete materials are more likely to be used in kindergarten and primary classrooms, and are typically used for remediation in junior and intermediate classrooms.

Additionally, the data gathered from educator interviews suggest that teachers have set beliefs about the use of concrete materials, which are linked to their views of mathematics itself (GECDSB Math Task Force Report, 2016). The following statements were noted in anecdotes collected from educators across the GECDSB:

- Concrete materials are great but kids can use them forever, so it is best to just use numbers.
- Math manipulatives are not appropriate for the kind of mathematics we are doing in intermediate.
- There is no way to represent the intermediate mathematics using concrete materials.
- Concrete materials are great for young children to learn some of the basics.
- Math manipulatives are good as a start, but then we need to get to 'the math.'
- Math manipulatives are great for kids who are struggling, but they are time consuming for the rest of the class.
- The manipulatives are there if they need them, but my kids prefer not to use them.

The belief sets were summarized in a GECDSB Math Task Force addendum document and can be summarized in the following statement: *concrete representations are not real math*. Real math is represented by abstract symbols (GECDSB, Math Task Force Report, 2016). Examining and challenging these belief sets are important as they underpin educator practice.

Literature Review

The Ontario Curriculum for Mathematics maintains that the use of multiple representations is important for learning. Concreteness Fading helps to support the connection between concrete, visual, and abstract representations. Fyfe et al. (2014), explain Concreteness Fading by using the example that the "quantity 'two' could first be represented by two physical apples, next by a picture of two dots representing those apples, and finally by the Arabic numeral 2" (p. 11).

Multiple studies identify that in contrast to models which encourage the simultaneous presentation of mathematical examples using concrete, visual and abstract models, Concreteness Fading involves the presentation of multiple models in a gradual specific progression (Fyfe et al., 2014). The work of Jerome Bruner ascertains the gradual and explicit fading, through which learners are able to strip the concept of extraneous, concrete properties and grasp the more portable, abstract properties (Bruner, 1966).

Concreteness fading assumes that learners can easily comprehend the concrete materials, are unfamiliar with the abstract materials, and have a certain level of "readiness" to learn. If learners lack sufficient prior knowledge to understand the concrete materials, then a fading progression may be too premature. On the other hand, if learners already have a sophisticated understanding of the abstract material, then they may benefit from working directly with the symbolic representations.

There is little in research to identify an ideal timeline for Concreteness Fading. Applied to the daily practice of educators it is necessary to uncover if concreteness fading happens over hours, days, weeks, or years.

Sarama and Clements (2016), write about confronting findings regarding the usefulness of manipulatives for learning mathematics—sometimes children who used manipulatives score worse on the tests. The authors speculate that, "One possibility is that instruction does not adequately promote *connection* between children's representations based on manipulatives and those based on paper and pencil (e.g., Carnine, Jitendra, & Silbert, 1997; Sherman & Bisanz, 2009)." As much as symbols may be learned through rote, use of manipulatives does not secure conceptual understanding. Manipulatives do not carry the meaning, but this meaning needs to be explained to children, they need to see them as representing mathematical ideas. Also, manipulatives do not have to be physical objects—they could exist on the computer screen. Manipulatives are useful because they are manipulable, not just because they are physical. That is why manipulatives on a computer screen could be equally useful; although not physical, they can be manipulated! Sarama and Clements (2016) further warn educators that manipulatives should not be used as toys, instead, the children should be clear that they are tools for thinking, not for playing.

Manipulatives need to be recognized as representations of abstract concepts (Marley & Carbonneau, 2015). They should be used with children who are ready to recognize such representation. Teachers should monitor how students use manipulatives and provide them with enough guidance. The research points that,

> *…with learning outcomes associated with retention and problem solving, older students benefitted from manipulatives more so than younger students and high levels of instructional guidance were more effective than lower levels of guidance. In addition, math topics that are more readily represented by objects such as fractions and place value had larger effect sizes relative to more abstract concepts such as algebra.* (Marley & Carbonneau, 2015, p. 418)

Concreteness fading is an effective instructional method with manipulatives. Students learning fractions, for example, may start with plastic pieces, which could be when appropriate replaced by pictures of these pieces. Students who could identify fractions in these forms, could then move to their symbolic representation. Teachers should make connections and commonalities between these representations explicit, but should not keep their students in the concrete phase unnecessarily long. While concrete material may interest students more than other representational forms, Day, Motz, and Goldstone (2015) highlight that too much detail may improve comprehension but deter transfer.

> *It is those very same qualities that are so beneficial in the learning of new material—concreteness, familiarity, personal relevance—appear so detrimental to the* ***generalization*** *of that knowledge….Simply adding richer meaningful content to in-class examples may make intuitive sense, and may have immediately obvious benefits in terms of student engagement and comprehension. But as the results of our experiments make clear, these short-term benefits seem to come at the cost of students' long-term ability to apply their knowledge. In other words, comprehension and transfer do not go hand-in-hand. (p. 2 & p. 11)*

After reviewing literature, Laski, Jordan, Daoust, and Murray (2015) proposed the following to educators:

> *(a) Use a manipulative consistently, over a long period of time;*
> *(b) Begin with highly transparent concrete representations and move to more abstract representations over time;*
> *(c) Avoid manipulatives that resemble everyday objects or have distracting irrelevant features; and*
> *(d) Explicitly explain the relation between the manipulatives and the math concept.* (Laski et al., 2015, p. 2)

These researchers use the Montessori approach to contextualize these principles. Since the Montessori levels encompass a three-year mixed age groups, in their early childhood classrooms there are 3-6 years old children. Such grouping is beneficial, as children move from concrete to abstract over the extended time period. Following the principles of Montessori education, educators should make sure "that the same or similar manipulatives are used over a long period of time and that instruction progresses from concrete to abstract representations…to ensure that the manipulatives used in instruction have few distracting features, teachers could minimize or eliminate the use of theme-based manipulatives (e.g., bug or teddy bear counters) and move instead toward using one or two general manipulatives (e.g., Cuisenaire rods, counting chips) for mathematics activities" (Laski et al., 2015, p. 7).

Assumptions about Concreteness Fading

Based on the growing body of research, Concreteness Fading does not align to a rule-based approach to mathematics learning. Inherent to the existing research as well as the dispositions of the educators in this inquiry, there are assumptions that were identified by the Eastwood Inquiry team. They include:

1. *Students construct their own understanding of mathematics, which cannot be transmitted;*
2. *Concrete, visual, and abstract representations are all valid and valuable;*
3. *There is a desire to support students' ability to understand abstract mathematical representations and this is done through connected experience with concrete and visual representations;*
4. *The mathematics in the Ontario elementary curriculum is grounded in concrete mathematics of our world and therefore the concepts, processes, and procedures can be represented using concrete materials;*
5. *Concrete or visual representations are no less 'mathematical' than abstract representations.*

Data Collection

Data were collected from students, teachers, and administration throughout the process. The data included interviews, reflections, lesson and unit plans, meeting notes, student work samples and student conversations gathered through notes or video. The data served to inform the ongoing process of developing an application of concreteness fading in the elementary classroom.

Planning Instruction using Concreteness Fading

Concreteness fading is described as a pedagogical approach, but for classroom teachers this approach needed to be translated into a familiar instructional framework. It was clear from the research and the early attempts at concreteness fading that it could be an approach embedded into and across lessons, units, and grades.

The educators at Eastwood Public School created a 3-stage planning structure. After many iterations, the team developed a planning framework which assisted in unit planning (see Appendix 1; Figure 1). Units were typically 4-6 weeks in length. The planning structure identifies the three stages of Concreteness Fading. Along the bottom of the template is a timeline. Depending on the grade level expectations for the mathematical concepts as well as the prior knowledge of the students, their planning would start or end at varied points along the timeline. For example, some mathematical concepts in Kindergarten are nearly entirely concrete or visual. In addition, intermediate students may be extending their investigations of a mathematical concept that was first introduced in the junior grades and begin at the later concrete stage.

Mathematical Concept Big Ideas Curriculum expectations Skills and Strategies				
	Conceptual Concrete Task or Mathematical Provocation	CONCRETE	VISUAL	ABSTRACT
Tasks Tools and Representations Discourse Prompts and Questions				
Assessment Points				
Timeline				

Figure 1. A planning framework used for unit planning.

It is important to note there is a greater length of time spent on the concrete phase. The data gathered from students identified that they spent greater time at the concrete stage, less time in the visual stage, and considerably less in the abstract stage. Student data demonstrated that once students were able to accurately and confidently reason through and solve a variety of problems using concrete and visual representations, they were quickly able to develop a mathematical procedures, rules, sequences or apply the use of symbols or notation with meaning and accuracy.

The unit of study generally began with a concrete conceptual mathematical task that set the context for the rest of the lessons. This task, or mathematical provocation, was a means of assessing student understanding. It was an investigation that supported students to use their prior knowledge to solve the problem. The task always promoted the use of concrete materials but did not limit student thinking to only this representation. The task typically underpinned the tasks and lessons of the rest of the unit and was revisited at other points in the unit and at the end. Students were able to reflect at the end of a unit of study about how their understanding changed from the initial task through the entire unit as evidence of learning.

Key Learning: How do I make this concrete?

Professional learning was key in supporting the work of Concreteness Fading across the school. The depth of understanding on the topic ranged across the school. Early discussions and moderation of student work helped to pinpoint concrete materials as a starting point for professional learning. In one of the early meetings, teachers were reviewing the Ontario curriculum and examining it through the lens of Concreteness Fading (see Figure 2). As part of the discussion one of the teachers remarked:

> *Every time I teach a concept, I have to ask myself, how do I make this concrete. And to be frank and honest, this is difficult, because representing the concept concretely is not how I understand the Mathematics. It certainly is not how I taught the math. ~Eastwood Teacher*

> *This makes me reflect and question if I – if 'We', really understand the mathematics because it seems to me that someone who has a complete understanding, someone who can claim to be mathematically proficient (National Research Council, 2001) can represent his/her mathematical reasoning in multiple ways. If we can only represent out thinking one way, then I suppose, our understanding is incomplete. Not incorrect, but incomplete. I see the question, "how do I make this concrete?" as a challenging professional learning opportunity that will serve to enhance the practice of every educator. If each of us asked this of ourselves, our understanding of mathematical concepts would grow and deepen. ~Principal and Facilitator Reflection*

Figure 2. Educators mapping curriculum expectations with concrete-visual-abstract phases.

This disclosure was thought-provoking and practice-changing. The educators revisited their guiding assumptions and added the following:

> *6. Teachers are able to represent mathematical concepts and procedures concretely, visually, and abstractly.*

Sub-stages of Concreteness Fading

Concreteness Fading is a pedagogical approach that supports the development of mathematical concepts. When educators understand the development of mathematical concepts, Concreteness Fading emerges as a natural fit, because it aligns to this development. Within each of the stages of Concreteness Fading, there are sub-stages.

Documentation of student learning revealed identifiable sets of behaviours that students demonstrated as they worked through the stages. In addition, there were specific pedagogical moves that emerged from the lesson and unit data that supported this students' progress.

The stages of Concreteness Fading included an *early*, *working on it*, and *transition* sub-stages.

Within the *Early Phase* student thinking was described as exploratory, tentative, based on prior knowledge. Students drew on familiar ideas and demonstrated partial understandings. For example, a young student learning about addition and subtraction of single digits at the early abstract phase would use familiar notation to communicate her thinking.

At the *Early Phase* of each stage, the pedagogical moves of the educator include modeling and very explicit instruction that helps students refine their understanding of the representations and tools. Another example might be that students at an early visual phase would communicate their thinking by drawing the exact concrete material that they had been working with.

During the *Working on It* phase students demonstrate confidence with the tools and representations of the stage and are able to use multiple variations. For example students learning about operations with fractions within the concrete phase use pattern blocks, square tiles, and fraction strips confidently. In this phase students would choose the representation that most efficiently helped them with the task. A student explained, "*I chose to use the pattern blocks because they show equivalence of thirds and sixths nicely.*"

At this phase, educators introduce multiple tasks and tools. In addition, the tasks become increasingly complex so as to push students to refine their mathematical thinking by choosing ever more efficient tools and representations. A teacher remarked, "*I see that this small group has got the concept concretely. Today I gave them a task with larger and more numbers, because I want them to move from concrete to visual. Immediately, they asked if they could draw their representations instead of building it because the numbers were just too big to build.*"

Students naturally moved through all three stages because they would seek out more efficient ways to solve problems. However, if the tasks remained at a consistent level of difficulty and complexity students would generally remain in that stage. In some cases, depending on the concept, this is exactly what we may want children to do. Some concepts require extensive time and experience in that stage before students themselves are developmentally ready to move on. If it was appropriate, as determined by the curriculum and the development level of the child, for students to move on through the Concreteness Fading stages, teachers needed to construct learning opportunities that pressed them to move.

In the *Transition Phase* students move in and out of the previous and next stage, and their representations sometimes lack clarity and efficiency. Students were efficient in their previous stage and trying a new representation requires additional time for sense-making. In order to promote transition, teachers invited students to investigate more efficient representations. In addition to increasing the complexity and difficulty of the mathematics tasks, teachers used questions to stimulate students to move to the next phase:

- *How can you use your knowledge of math symbols and notation to help you communicate what you have in your illustrations?*
- *Can you use visual images to help you solve this problem?*
- *You have built your representations using lots of different manipulatives, do you think you could draw what you built? Which do you prefer and why?*
- *What is it about today's problem that makes you want to find a more efficient way to show your thinking?*

Points to consider

<u>Concrete means *concrete.*</u> For our youngest learners the manipulative may already be an abstraction. For example when using a rekenrek with kindergarten students, it may be difficult for some to see how the beads represent the number of friends and can be counted. Instead, students need to count the actual friends.

<u>Concreteness fading for *student* learning.</u> Concreteness fading is about students working through the stages to construct their own mathematical understanding. A teacher who models lessons and demonstrates representations of the mathematics concretely, then visually, and then abstractly, is not engaging in a constructivist form of learning. In a constructivist model, the teacher's role is very complex. The teacher must create learning opportunities that encourage students to construct their understanding.

Things We Noticed

The following is a collection of observations and reflections from the teacher and student data. Each of these are areas for further or more intensive investigation and help to inform future work.

Mistakes took on a qualitative change. The mistakes that students were making were qualitatively different. Many times students make 'silly mistakes' which are calculation or procedural errors, thought of as silly because they omitted a step in the procedure and their solution was unreasonable. By engaging in this process, students' adaptive reasoning and conceptual understanding improved and there were less of these errors. The observed student errors were now over reaching a conjecture… *I developed this rule and I think it works all the time.*

Intentional selection of manipulatives and tools. The selection of tools and manipulatives need to support the fading process. Each manipulative has properties that either open or constrain thinking. It is vital that teachers are aware of these properties and make intentional, methodical selections. At time, it is appropriate for student to self-select the tools. At other times, teachers need to choose the tool because it drives the learning. For example, using connecting cubes (which snap together) to model a task about mean, helps students to see the procedure concretely.

Teachers need to understand math concept development and connections. In order to best support concreteness fading, teacher need to know how the mathematics concepts develop and how they connect.

Teachers need to have a thorough understanding the Ontario Curriculum and the continuum of learning. The Ontario curriculum needs to be understood in the context of mathematics development and connections. The Ontario curriculum can and should be explored through this lens.

Teachers need to know their students as math learners. A thorough understanding of the mathematics content helps teachers to better understand their students as learners. This knowledge of students is imperative when planning precise and focused instruction.

Amounts of time. Concrete stage is lengthy; Visual is less; Abstract representations can go very fast.

Conceptual Concrete Task. It was important for the initial task given to students to be contextual, sensory, kinesthetic, and tactile because this task served as a catalyst for the concept development and as an underpinning for all other tasks in the pathway/unit. It is imperative that we begin with concrete representations and concrete conceptual tasks when a concept is brand new or when it is developmentally appropriate.

Extending students thinking through questions

Within and across stages there are key points at which we can extend student thinking.

Concrete Stage

What other tool can you use to represent your thinking?
What would it look like if we used this manipulative?

Moving to Visual Stage

How could you represent your thinking using pictures?
Could you draw the math?
Can you use an illustration to represent your thinking?

Refining Thinking in the Visual Stage

Is there another visual tool that you can use to represent your thinking?
Is there another visual symbol that you could use to represent your thinking?
What would it look like if we used this visual manipulative?
What would it look like if we used this virtual manipulative?

Moving to the Abstract

How can you use your knowledge of numbers and symbols to represent your thinking?

Refining Thinking in the Abstract Stage

How can we make this more concise?

How can we simplify this idea?

How can we apply the big ideas of mathematics to clarify our thinking?

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Appendix: Finding arithmetic mean by fading concrete representations

*This series of tasks is constructed to highlight the concreteness fading of one mathematical procedure – finding arithmetic mean. It is not an accurate representation of a complete unit but only a simple example.

Task 1 – Early Concrete Stage

On Saturday morning Nick's mom bought for him and his two sisters Timbits. Nick ate 5, Hannah ate 3, and Jill ate 4. Nick's mom wants to figure out how many on average the kids ate (the mean value) so that she can buy enough Timbits next week when their cousins come over.

> Notes: Students will model the problem with Timbits. Use the concrete, sensory and spatial value of the model to redistribute the Timbits to find a mean of 4.

Task 2 – Later Concrete State

Invite students to model the previous task using connecting cubes and identify the mean.

The next Saturday morning Nick's mom bought Timbits again. This time Nick's two cousins were there as well. Based on the mean that she calculated from the previous week she figured she had to buy a 20pack. Use the connecting cubes to model the problem and see if the 20 Timbits worked out. Can you explain the Mom's thinking?

> Nick ate 5; Hannah ate 3; Jill ate 6; Sima ate 2; Henrick ate 4.

Task 3 – Early Visual Stage

Invite students to model the previous task using a graph or virtual manipulative and identify the mean.

Task 4 – Later Visual Stage

The Pike City Jaguars are a hockey team with 9 players. They rotate positions on the ice. The team is interested in finding the mean scoring for the year. Here are each of the players and their data.

Hassan	9
Kelly	6
Gemma	8
Winston	3
Graham	11
Bailey	4
Pat	3
Keshawn	6
Andrea	4

Task 5 – Early Abstract Stage

Invite students to model the previous task using their knowledge of numbers and symbols. (They will develop their own procedure)

Task 6 – Later Abstract Stage

Hassan	9	Kyle	6
Kelly	6	Sharon	7
Gemma	8	Angeline	5
Winston	3	Mike	4
Graham	11	Fouada	3
Bailey	4	Brenda	5
Pat	3	Bill	1
Keshawn	6	Clara	2
Andrea	4	Anna	3
Shelley	3	Chad	5

Test and refine the procedure. The class has enjoyed eating Timbits. Next week we need to buy Timbits for the school. We can use the class mean to help us extrapolate that data to the whole school. Here are the class data. Apply your procedure. What is the mean? How can you refine your procedure? If there are 326 kids in the school, how can you use the class mean to help you predict how many Timbits are needed for the entire school?