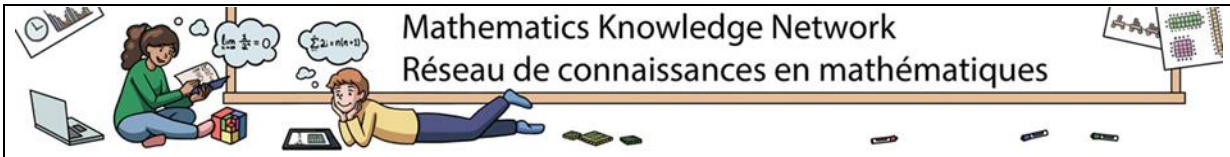
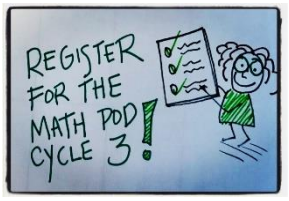



# THE MATH POD 3

During the four weeks of April 2018, educators covered a lot of ground as they were led in the professional learning by Cathy Fosnot and the Ministry of Education team of facilitators. As in the previous rounds of The Math Pod, educators could listen to the weekly radio shows on voicEd Radio (live or as podcasts) and participate in Twitter chats. Except for the new topic—moving from arithmetic to algebra—the novelty was in educators having the opportunity to register for free onto Cathy Fosnot's online platform, *A Personalized Professional Support System™ (P2S2)* where they could participate in the online PLC and access for free videos of children and teachers, screencasts by Cathy, and assignments/investigations built into the platform. As a community, they could participate in online discussions.




While most of the 143 educators who registered for The Math Pod 3 were from Ontario school boards, we also had 9 from Newfoundland & Labrador, one from the Prince Edward Island, and 16 from the United States. Seven educators came from two Ontario French school boards. Ontario educators came from 26 secular and 14 catholic school boards. This wide spread participation created a fertile ground for cross pollination of professional learning across more than 50% of Ontario public school boards!

Even before the first session of The Math Pod 3, the Twitter community was buzzing with anticipation. From March 20-April 30, 264 the #MathPod Tweets were seen by about 64,000 people, and #MathPod had 754 Followers (increase of 31% after the second cycle). Only in April, #MathPod attracted 101 new followers and had 1,311 profile visits.

On P2S2, educators contributed 89 responses to Cathy Fosnot's questions. The radio show had on average 45 listeners and 50 downloads per week. The breakdown of downloads is as follows:

- Episode One— April 4th: Arithmetic to Algebra, downloaded by 88
- Episode Two— April 11th: Leveraging the Mini-Lesson, downloaded by 47
- Episode Three— April 18th: Variation and Representation, downloaded by 31
- Episode Four—April 25th: Representation, the Reversal Error, and documenting our own growth on a landscape of teacher change, downloaded by 32.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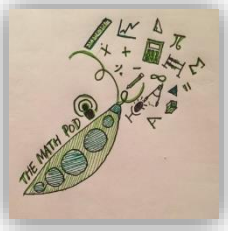
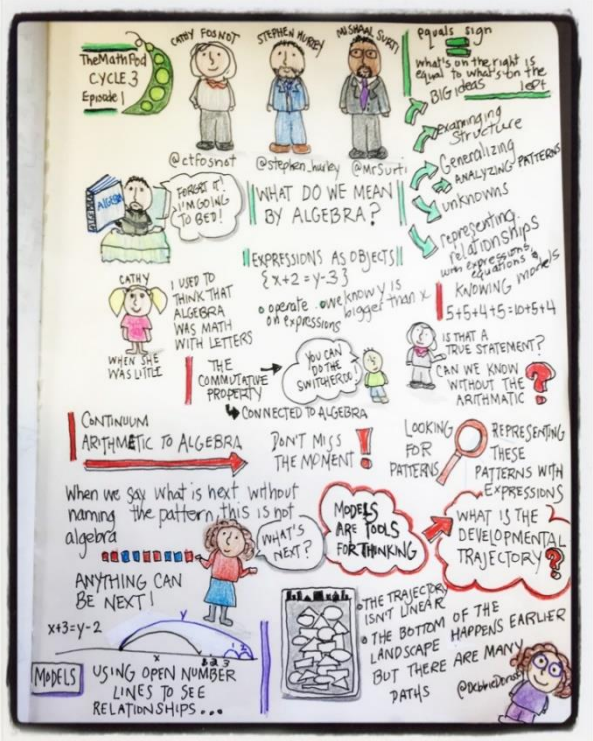



*"I believe we have a multimedia extravaganza going on here,… with Tweets, VoicEd Radio, and an online platform full of video of vibrant math workshops in action!" (Cathy Fosnot)*

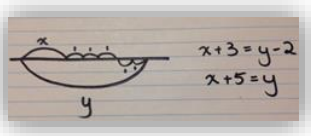
# Arithmetic to Algebra: The first episode of The Math Pod 3 and highlights from Stephen Hurley's conversations with Cathy Fosnot.




In this episode, Cathy Fosnot addressed the following questions:

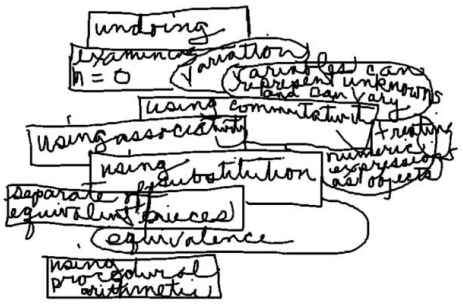
–What is mathematics and what is algebra, and what does it mean to do them?

–What does the development of algebra K-8 look like? What is the landscape?




Cathy: "Algebra is not about letters, it is about generalizing! In the equation **x+3=y-2**, we are representing the relationship between y and x. We can treat expressions (i.e., x+3 and y-2) as interchangeable objects because they are equivalent. For example, if I line them up on a double number line, I set them up as being equal and we can see how the variables vary, that **x+5=y**. Also, whatever the y is, if you take away 5, you get x. The beauty of treating an expression as an object is that you can just set up the relationships and operate on them. You don't always need to solve for the unknowns."

*"We see a lot by modeling. We can examine structure and regularity. The model becomes a tool for thinking and that's part of doing algebra, too!" (Cathy Fosnot)*




Cathy also shared a landscape for early algebra. Her landscapes are nonlinear developmental trajectories that teacher can use as a lens to monitor and document a child's growth and development as a mathematician. It is much more helpful for us as teachers to think about the developmental progression of strategies kids might use, the big ideas kids have constructed (the properties are big ideas), and the models they use as tools…

One early big idea is *equivalence*. The equal sign does not mean "the answer is coming." It means the expressions on each side of the equal sign are equivalent, even if they look different. It is more powerful to introduce variables in different ways, on both sides of the equation, not just by using an empty box (e.g., **5+☐=8**). *Variables can represent unknowns and they can vary*, is a big idea. When C is added to both sides of an equation, equivalence is maintained, and one can see that C can have many values and the equation remains true. Strategies, like using substitution and exchanging objects that are equivalent, are also important. Using equivalence and separating off equal amounts to simplify computation, is an important strategy. For example, **5 + 5 + 9 = 9 + 10** is a true statement because of the commutative property and the fact that 5+5 is interchangeable for 10. No arithmetic is needed!

*"Why is it important for elementary teachers to think about algebra? It is because there are so many moments when children's arithmetic could be extended to algebra, but we miss these moments and focus only on solving the problems at hand with arithmetic." (Cathy Fosnot)*

# Leveraging the Mini-lessons: The second episode of the Math Pod and highlights from Stephen Hurley's conversations with Cathy Fosnot.

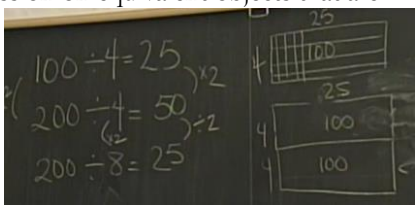

In this episode, Cathy Fosnot addressed the following topics:
– Doing mini-lessons through the grades for computational fluency and algebraic reasoning with strings of related problems
– Documenting the learning using the algebra landscape.

Stephen: "The mini lessons are really important for discovering big ideas, but where do they fit in the context of the math classroom?"

Cathy: "More often than not, you start a math workshop with a 15min mini lesson. …The teacher usually has kids clustered around her in a meeting area, and she uses a string of related problems doing one at a time (usually comprised of 5-6 problems). It encourages students' thinking."

Stephen: "How do you develop a string? Are you looking at the landscape while doing so?"

Cathy: "The landscape is critical. The string is crafted not just on one idea though but to ensure that we get certain conversations to happen. So, if the conversation is about treating expressions as objects, other ideas might also come up, but at a minimum a discussion on equivalent objects that are interchangeable will come up."




Stephen: "What do you mean by 'arithmetic being extended to algebra'?"

Cathy: "Focusing on only arithmetic and the answer, we often miss the algebraic moments. The big ideas go by us—we do not do the extension. And, the extension is about generalization. Algebra is about symbolizing, yes, but it is about equivalence, variation, structuring, and generalizing. …It is much more than letters!
When we divide a fraction by a fraction, we can use equivalent ratios (objects that are equivalent). For example, if we want to divide $\frac{3}{4}$ by $\frac{2}{3}$, we can just substitute $\frac{\mathbf{3}}{\mathbf{8}}$ divided by $\frac{\mathbf{1}}{\mathbf{3}}$, which is equivalent to $\frac{\mathbf{9}}{\mathbf{8}}$ divided by 1…All we are doing is taking objects and substituting or exchanging one equivalent object by another. Hidden here is the algorithm that we were taught as a procedure: *invert and multiply*. The ratio remains the same when both terms are multiplied or divided by the same number. We have only scaled the ratio up or down. In the prior example, we just divided both terms by 2, and then multiplied both terms by 3. But instead of working with a procedure, we used algebra and just replaced one object by other equivalent objects, working through to replace the divisor by 1."

*"'Relationship,' is such a powerful mathematical concept, as well as a human concept!" (Stephen Hurley)*

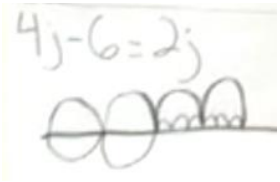
*"In the landscape, we see the big ideas of equivalence and of treating expressions as objects." (Cathy Fosnot)*

*"When I look at the landscape, I wish that I had that when I was in the classroom." (Stephen Hurley)*

# Variation and Representation: The third episode of The Math Pod 3 and highlights from Stephen Hurley's conversations with Cathy Fosnot.


In this episode, Cathy Fosnot talked about the importance of anticipating what children will do and analyzing their work using the landscape.

Cathy: "If I say to you, 'A frog went 4 equal jumps and back 6 steps,' you do not know how big a jump is. But if I say that '**4j – 6 =2j**,' we force the value. Try, instead of using a procedure, to use an open number line to model the problem…The minute you do that, the 2js on the top and bottom cancel, and it is clear that 2js are equal 6. And, if **2j=6**, then **j=3**."



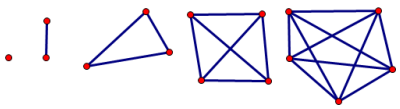

Stephen: "The open number line does not have a scale… there is a real freedom in that!"

Cathy: "An open number line allows you to model the problem. It's a tool. You do not have to worry about counting the ticks (as in a common number line), you do not have to worry about making everything exact—you are just using it as a tool for thinking. You are structuring the problem with this tool in order to examine it. It allows you to see how one expression varies in relation to another. …You are treating expressions as objects that are interchangeable, because they are equal, a powerful big idea when we get to systems of equations!"

Stephen: "In Ontario, patterning and algebra are the same strand, but teachers teach them separately. Why are they put together?"

Cathy: "Algebra to me is about structuring; it is both about uncovering structure as we are about to solve the problem, but it is also about looking at patterns, structuring and analyzing them, trying to find ways to represent the structure (e.g., the Handshake Problem). …Early on, we invite the children to develop number structures, and to develop a number system. And then, we move into algebra—we invite them to structure patterns and represent relationships in symbolic generalized ways to see how one thing varies in relation to another, and how objects can be equivalent even if they look different. And, how you can use this structure to substitute and exchange for another structure."

Handshake Problem

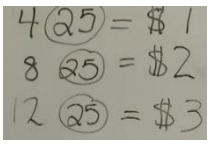


1 person, 0 handshakes
2 persons, 1 handshake
n persons, $\frac{n*(n-1)}{2}$ handshakes

*"This is why I want to transform a math classroom into a math workshop. I want children to have the opportunities to do what mathematicians do! Otherwise, we kill it for them. There is no beauty or invention….Think about the secondary school algebra, where you have to solve something. What you are really doing is substituting and exchanging equivalent pieces. You are constantly juggling these objects. .. When we do these things with numeric expressions, with younger children, we allow children to see the beauty of doing mathematics with symbols." (Cathy Fosnot)*

# Representation and the Reversal Error: The final episode of The Math Pod 3 and highlights from Stephen Hurley's conversations with Cathy Fosnot.


In a P2S2 video clip, a teacher asks the children: "If there are 4 Quarters, how many Dollars are there? And, if there are 8 Quarters, how many Dollars are there?" Then she records students' thinking in the following way:




Cathy: "Would there be more algebra if the teacher used letters for coins, for example Q and D?"

**4D** means "four times any number of Dollars." And, **4D=Q** represents the relationship between any number of Dollars and any number of Quarters. This relationship is seen in the Ratio Table. We could also graph this relationship, putting **D** and **Q** on the axes. This expresses a linear relationship, where 4 is a slope. If the teacher from the beginning wrote **4Q=D**, she would create a stage for the reversal error to develop.

| Q | D |
|---|---|
| 4 | 1 |
| 8 | 2 |
| 16 | 4 |
| Q | ¼ Q |

**Therefore, D = 1/4Q**

*"It is an amazing journey from arithmetic to algebra!" (Stephen Hurley)*

*"So many people think that algebra has to have letters, and it doesn't" (Cathy Fosnot)*

*"Years ago it was OK if you did not want to take Algebra courses in high school; if you weren't going to pursue math on the college level, it wasn't so critical. But, in today's cyber world, there is nothing more important than math." (Cathy Fosnot)*

## Acknowledgements


We thank all educators who participated in the Math Pod 3. Big thanks to educators who contributed to the podcasts: Mishaal Surti, Kathy Prince, and Debbie Donsky (Student Achievement Officers, Ontario Ministry of Education), and Ryan Tackaberry (Grade 6 Teacher). All sketchnotes were created by Debbie Donsky.

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